Contemporary Model Life Tables for Developed Countries

An Application of Model-based Clustering

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Abstract

Using the statistical technique *Model-based Clustering* (Fraley and Raftery, 2006) we identify five typical age patterns of mortality in 844 life tables from the Human Mortality Database (University of California, Berkeley and Max Planck Institute for Demographic Research, 2009). Time rather than geography is the most important dimension along which the Human Mortality Database life tables are clustered. Using each pattern as the basis for a 'family' in a traditional system of model life tables, we create a one-parameter model to generate 'levels' of mortality within each family. The result is a new effectively two-parameter system of model life tables for the countries and time periods included in the Human Mortality Database. We demonstrate the use of this model life table system to extrapolate full age patterns of mortality from age-restricted mortality indicators such as ${}_5q_0$ or ${}_{45}q_{15}$. We conduct an out-of-sample validation using life tables from both the Human Mortality Database and the WHO life table database (World Health Organization, 2010). A friendly `R` package that implements the model life table system calibrated to the Human Mortality Database life tables is available by request.

Keywords: *Model Life Table, HMD, Model-based Clustering, Mortality.*

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1 Introduction

1.1 Motivation

Absent high quality vital registration systems that could accurately describe the age pattern of mortality, many countries must rely on indirect estimation by using systems of model life tables. Model life table systems represent typical age patterns of mortality that can be used for many purposes including to extrapolate complete age patterns from age-restricted indicators of mortality such as child (${}_5q_0$) or adult (${}_{45}q_{15}$) mortality.

Existing model life table systems were generated using data from restricted geographical locations and time periods, with the newest model systems dating back some 30-50 years. In some instances, these systems are unable to reflect contemporary mortality experiences including extremely low childhood mortality observed in some contemporary developed world settings (Coale and Guo, 1989; Wilmoth et al., 2009) or elevated adult mortality resulting from unusual causes such as HIV.

Building on earlier work (Clark, 2002; Clark et al., 2009), we aim to identify commonly observed age-patterns of mortality in the Human Mortality Database (University of California, Berkeley and Max Planck Institute for Demographic Research, 2009) and build an easy-to-use model life table system based on the observed mortality patterns. We identify typical age patterns by using a model-based clustering method. Using each pattern as the basis for a 'family' in a traditional system of model life tables, we create a one-parameter model to generate 'levels' of mortality within each family. The result is a new, effectively two-parameter system of model life tables for the countries and time periods included in the Human Mortality Database.

We begin with a brief review of the use and structure of existing model life table systems, followed by a description of the Human Mortality Database. We then present a detailed description of the model, fitting method and resulting 'families'. Finally, we discuss usage and validation of the system with an out-of-sample collection of empirical tables. We also include a discussion of future directions for this line of research including extending the method to empirical tables from the INDEPTH network of demographic surveillance sites, which include tables from Africa and high HIV prevalence settings.

1.2 Existing Model Life Table Systems

All model life table systems, including the one presented in this paper, are generated from analysis of a large collection of historical mortality profiles. Unlike mathematical models such as Gompertz (1825) or the all-age model advanced by Heligman and Pollard (1980) that depict the shape of human mortality with relatively few parameters, *empirical models* like the Coale and Demeny (1966) model life tables, UN Model Life Tables for Developing Countries (1982), model by Wilmoth and colleagues (2009) and relational models like Brass (1971) and Murray et al. (2003) and the system described in this paper contain a larger

number of 'true' parameters but are able to reflect the detailed variation in human mortality with a very small number of 'effective' parameters. This is because most of the parameters become fixed leaving just one or two that vary to capture the variation in human mortality, often a measure of child mortality.

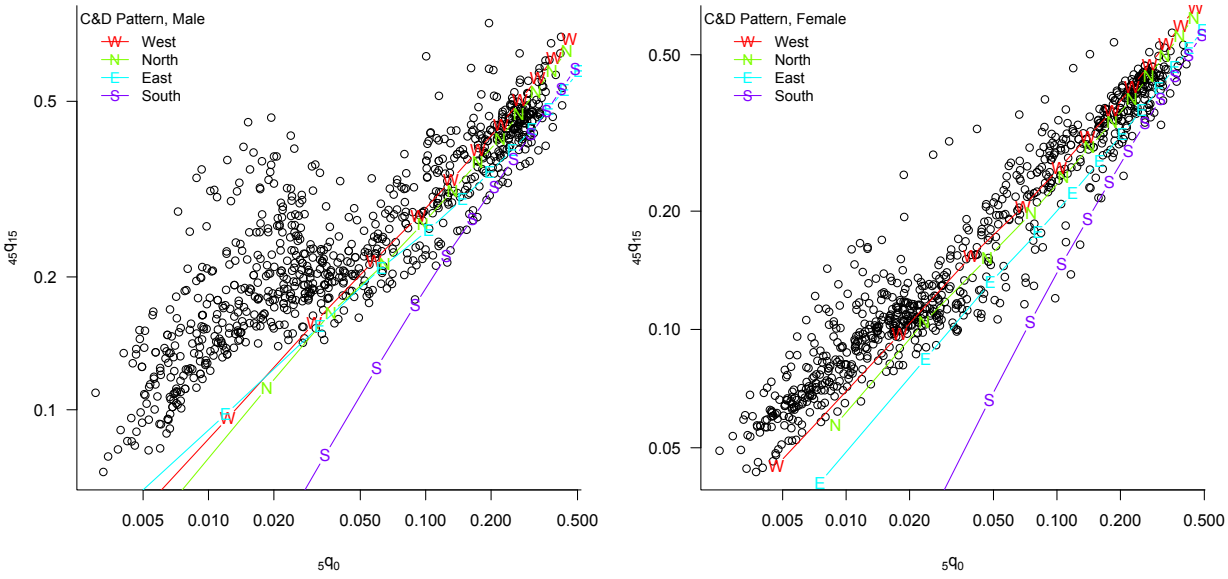
Because model life table systems are generated from a collection of empirical tables, they can only reflect the extent of mortality variation contained in the empirical data used to generate the system. For instance, the widely used Coale and Demeny and UN model life tables for developing countries cannot generate schedules for which child mortality is extremely low because that kind of mortality profile did not exist at the time those systems were created (Coale and Guo, 1989; Wilmoth et al., 2009). Figure 1 plots the relationship between adult and child mortality for the HMD collection (each black circle represents one table) and the patterns produced by these two systems. These figures show that those mortality profiles with extremely low child mortality are not well represented by any of the patterns in these two widely used systems. This issue is poised to become increasingly important as child mortality continues to fall throughout the world. Additionally, although not well represented in the data used to generate our system, no system to date has included adequate data from Africa or data that could reflect the mortality profile characteristic of high HIV prevalence settings. Thus, there is little reason to believe any system can accurately represent these profiles generated under a very different set of demographic and epidemiological conditions.

It is clear that creating a model life table system based on contemporary data that can reflect the current range of modern mortality experiences is an important improvement over current widely-used systems, but the work presented here has other advantages as well. Typically, the user must supply specific mortality measures like ${}_5q_0$ as input parameters. Our system allows the user greater flexibility in terms of the input parameters in that virtually any measure of mortality (${}_1q_0$, ${}_5q_0$, ${}_{45}q_{15}$, e_0, etc.) can be employed to select into the appropriate 'family' and subsequently the single-parameter model for selecting the appropriate 'level' ensures coverage of a wide range of mortality schedules.

2 Data

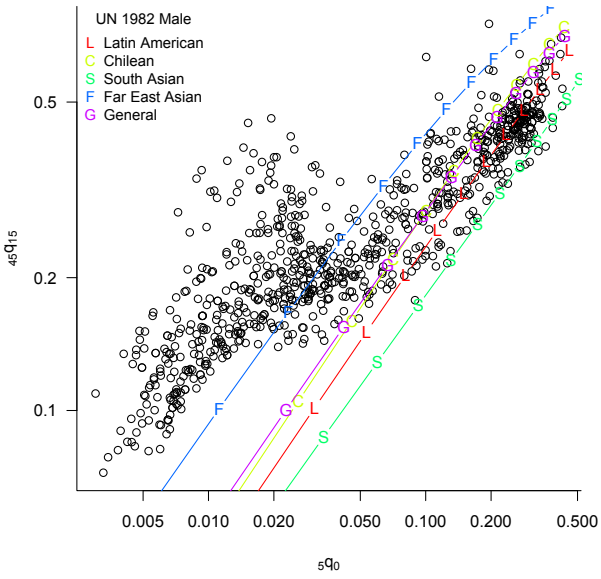
We identify similar age patterns of mortality within a collection of 844 (for each sex) period life tables from the Human Mortality Database. This collection is a publicly available dataset maintained by the University of California, Berkley and Max Planc Institute (www.mortality.org). This dataset has many advantages for generating a model life table system including the high quality of the data itself along with the standardized age and time interval formats.

The HMD contains mortality profiles from 37 mostly developed world regions with the earliest tables dating back to the mid 18^{th} century and the most recent from 2007. All of these life tables have been computed from directly-observed deaths and population counts without adjustment except at older ages (Wilmoth et al., 2007) and each table covers a

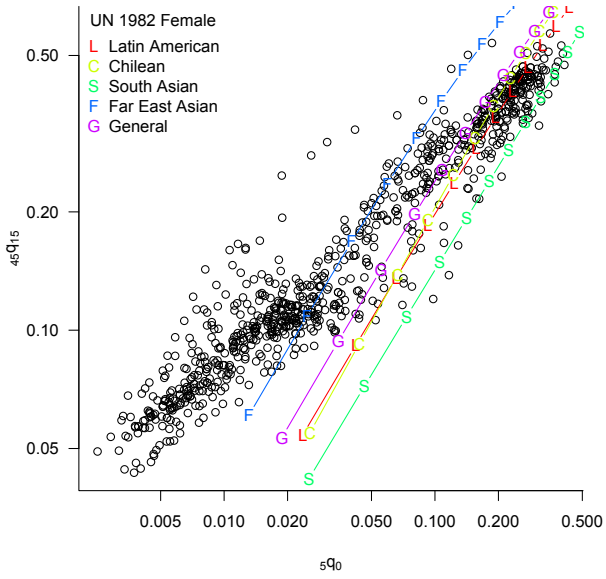


(a) C&D male

(b) C&D female



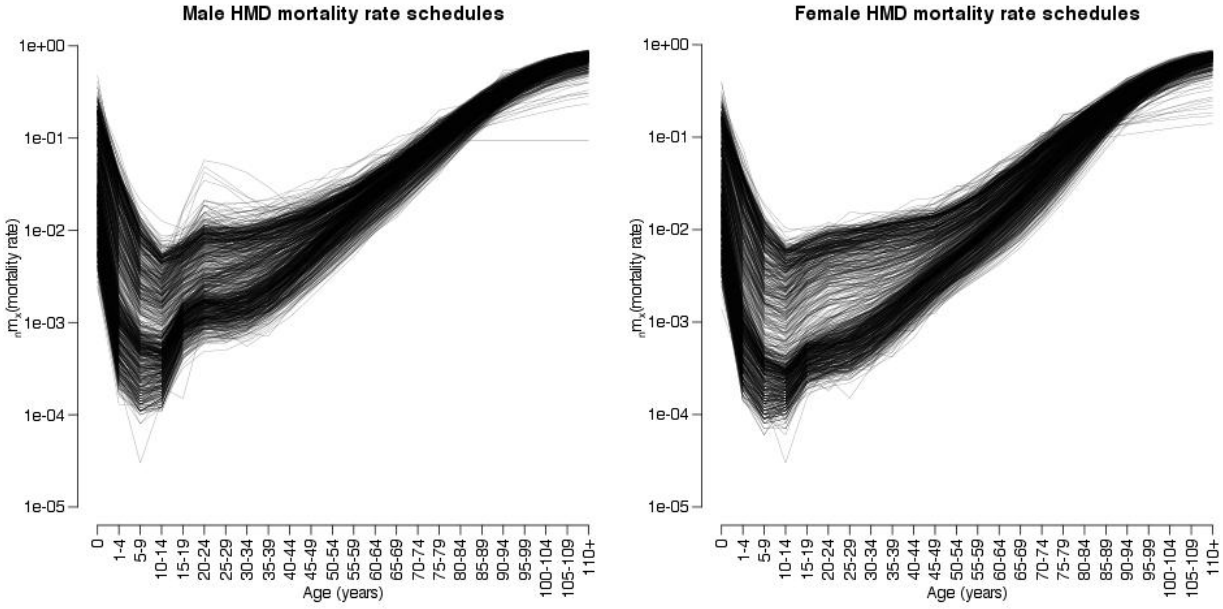
(c) UN male



(d) UN female

Figure 1. Relationship between child and adult mortality, observed HMD data (n=844) and Coale-Demeny and UN regional model life table patterns

5-year period. The life tables in this collection contain approximately 84 million male and female deaths and roughly 5 billion male and 5.5 billion female person years of observation. An important advantage of this dataset is that each table has an open age interval at 110+. This open interval means the system is able to produce complete sets of mortality rates with the same open interval, which exceeds that of other widely used systems.[1] We identify similar age patterns of mortality in the log mortality rate schedules ($_nm_x$), which are presented in figure 2.



(a) HMD data male

(b) HMD data female

Figure 2. Mortality Rate ($_nm_x$) Schedules in the Human Mortality Database (log scale)

3 Method

We begin this section with a discussion of the mortality model that we use to calculate a complete mortality rate schedule. We then describe the components of the model in two broad steps that correspond to the two parameters of the system - identifying 'families' with cluster analysis and defining 'levels' within 'family' with a one-paramter model.

3.1 Mortality Model

3.1.1 Motivation

The motivation for this model is two-fold. The model must represent mortality age patterns in a parsimonious way that helps identify regularities among possibly many empirical age

[1] In the widely used current systems the open age interval is typically 80+ or 85+.

patterns, and it must be able to represent a range of model mortality patterns based on the common patterns that emerge from the empirical data.

The general form of the model will be to represent a mortality age pattern as the weighted sum of two or more independent, age-varying components that represent the age-varying nature of the mortality schedule. To this we add a constant at each age to take into account the non-age-varying level of the mortality schedule. Any remaining differences between the modeled and observed age patterns are captured with a residual term.

The independent, age-varying components necessary for this model can be easily derived from a Singular Value Decomposition (SVD) of the matrix of observed mortality schedules. The resulting left-singular vectors[2] are the independent components we need, and they have the convenient property of encoding the bulk of the variance among the observed mortality schedules in a small number of components.

3.1.2 Model

Assuming 24 age groups (0, 1-4, 5-9, 10-14,... 110+), a 24 x m matrix $\mathbf{M}$ composed of m column vectors of age-specific mortality rate schedules can be expressed as a weighted sum of a number of components whose shapes encode the fundamental age pattern of human mortality and a wide range of variations on that:

$$\mathbf{M} = \mathbf{SB} + \mathbf{C} + \mathbf{R} \tag{1}$$

$\mathbf{S}$ is a 24 x n matrix whose columns are the n 'components' used in the model (derived from a SVD decomposition of all of the empirical mortality rate schedules). $\mathbf{B}$ is a n x m matrix whose columns are coefficients that multiply each component schedule contained in $\mathbf{S}$ to yield the age-varying component of each mortality schedule. $\mathbf{C}$ is a 24 x m matrix whose columns are constants that are added to the result of the multiplication to modify each mortality schedule in an age-constant way. Finally, $\mathbf{R}$ is a 24 x m matrix of residuals that account for the remaining difference between the modeled and empirical mortality schedules.

$$\mathbf{M} = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,m} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ m_{24,1} & m_{24,2} & \cdots & m_{24,m} \end{bmatrix} \tag{2}$$

$$\mathbf{S} = \begin{bmatrix} s_{1,1} & s_{1,2} & \cdots & s_{1,n} \\ s_{2,1} & s_{2,2} & \cdots & s_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{24,1} & s_{24,2} & \cdots & s_{24,n} \end{bmatrix}$$

[2]The left singular vectors are similar to the components derived from a principle components analysis.

$$
\begin{aligned}
\mathbf{B} &= \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,m} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n,1} & b_{n,2} & \cdots & b_{n,m} \end{bmatrix} \\
\mathbf{C} &= \begin{bmatrix} c_{\,\cdot\,,1} & c_{\,\cdot\,,2} & \cdots & c_{\,\cdot\,,m} \\ \vdots & \vdots & \vdots & \vdots \\ c_{\,\cdot\,,1} & c_{\,\cdot\,,2} & \cdots & c_{\,\cdot\,,m} \end{bmatrix} \\
\mathbf{R} &= \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,m} \\ r_{2,1} & r_{2,2} & \cdots & r_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{24,1} & r_{24,2} & \cdots & r_{24,m} \end{bmatrix}
\end{aligned} \tag{3}
$$

Ignoring the residuals, $\mathbf{SB} + \mathbf{C}$ represents the modeled mortality schedules. $\mathbf{SB}$ captures the age-varying component of the mortality schedules and $\mathbf{C}$ represents the non age-varying level of each mortality schedule.

If only one mortality schedule is involved:

$$
\begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_{24} \end{bmatrix} = b_1 \cdot \begin{bmatrix} s_{1,1} \\ s_{2,1} \\ \vdots \\ s_{24,1} \end{bmatrix} + b_2 \cdot \begin{bmatrix} s_{1,2} \\ s_{2,2} \\ \vdots \\ s_{24,2} \end{bmatrix} + \quad \cdots \quad + b_n \cdot \begin{bmatrix} s_{1,n} \\ s_{2,n} \\ \vdots \\ s_{24,n} \end{bmatrix} + \begin{bmatrix} c \\ c \\ \vdots \\ c \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{24} \end{bmatrix} \tag{4}
$$

The component vectors $s_{(\cdot,\cdot)}$ can be thought of as a new basis in age-space with the special property that most of the variance in the data is represented by a small subset of the these. The weights $b_{(\cdot)}$ determine a given point in this space, and then c adds a constant amount to the sum of the weighted components.

The effective parameters in this model are the b's, and as mentioned above, when only a small number of score vectors is necessary to account for the bulk of the variance in the empirical data set, then the number of b's necessary is small, making the model effectively parsimonious. Of course in reality the model is much less parsimonious because the component vectors $\mathbf{S}$ are parameters themselves, albeit fixed. Provided the original data set from which they are calculated is highly varied, they will be capable of representing a wide range of age variation and can be thought of as permanently fixed.

3.2 Identifying Similar Age Patterns of Mortality

We use a model-based clustering method to identify similar age patterns of mortality. Before the cluster analysis, we first reduce the dimensionality of the data using Singular Value

Decomposition of the empirical schedules of mortality. In order to maintain congruence of periods and locations between male and female schedules, we bind the male and female schedules and perform the SVD on the resulting 48×844 matrix. SVD decomposes a matrix into three smaller matrixes including one whose columns are orthogonal and point in the directions with most variation in the original space – the *left singular* vectors. These vectors concentrate the information in the original matrix into a smaller number of dimensions, and as a result the information in the original matrix can be represented by just the first few vectors. Thus, the dimensionality of the data can be reduced from 24 (or 48) age groups to just four or five component vectors. Figure 3 plots the first five component vectors from the SVD.

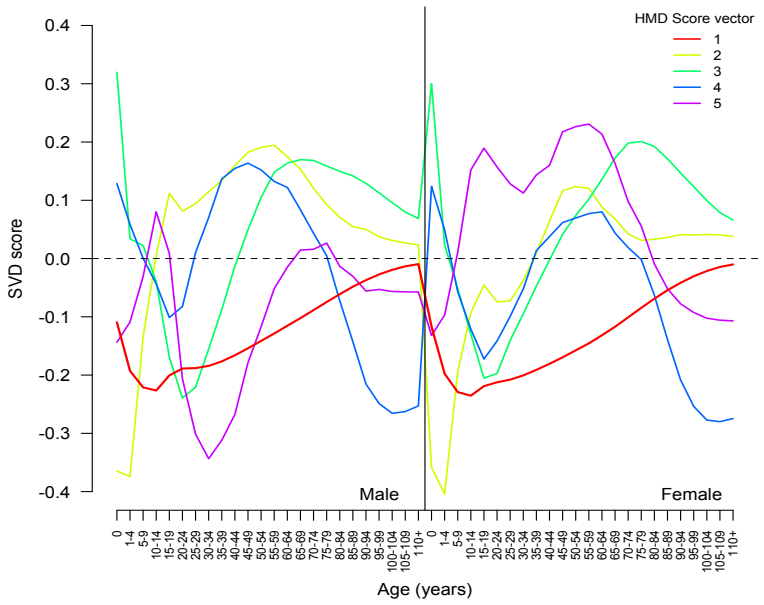


Figure 3. First five component vectors (left singular vectors) from the SVD of the HMD mortality rate schedules

This reduction in dimensionality is completed by regressing (simple OLS linear regression) each empirical mortality schedule on the first ten component vectors and storing the resulting coefficients and constants in a new data set. Keeping ten coefficients provides an opportunity to vary the number of components used in the clustering and choose the number that provides the best clustering.

With this new reduced-dimension data set, we use the model-based clustering method described by Fraley and Raftery (2006) to identify robust clusters. This technique is a fully automated, robust clustering method that identifies the number and shape of clusters that maximizes the bayesian information criteria (BIC). Model-based clustering puts cluster analysis on solid statistical footing and answers questions concerning not only the number and shape of clusters, but is also able to quantify uncertainty about the results. This method

yields both the BIC values for different numbers of clusters and the classification of the data using the number of clusters with the greatest BIC value.

A priori there is no objective way to choose how many of our new 'reduced' dimensions to include in the clustering, so clustering is performed on the reduced dimension data set using 2-10 dimensions, and the clustering classification from each is saved in a new data set. The best clustering is chosen by calculating a new fit metric, the "total deviation from median" (TDM). This is the sum of the absolute differences between each mortality schedule and the median of all mortality schedules in the cluster to which it is assigned. Lower values of the TDM indicate less variation among mortality schedules in each cluster, and consequently, the best clustering has the lowest overall TDM value.

3.3 Model Life Tables

Following the structure of existing systems, the one presented here is composed of 'families' with different 'levels' of mortality in each. Each 'family' is based on one of the clusters identified using the procedure described above. The structure of the system is presented in equation 5.

$$\text{(underlying family-specific age pattern)} + \alpha * \text{(family-age-specific deviation)} \tag{5}$$

where α varies to generate levels within a family.

3.3.1 Life Table Families

The underlying-family-specific age pattern of mortality for each family is identified by first obtaining the median set of coefficients for each family (cluster) and inserting them into equation 1 to obtain a 'family' pattern. The resulting cluster-specific median mortality rate schedules are the underlying mortality age profiles on which the model life table families are based $\mathbf{M}_f$, where f indexes the families.

3.3.2 Age-Varying Mortality Levels within a Family

There is variation in the overall level of mortality within each of the clusters that underly the model life table families. We include this variation in equation 5 with the family-age-specific deviation. This quantity is a weighted average of variation from two sources. The first captures age-specific variation within the family as the difference between the 97.5^{th} quantile for age group i and family f and the median value for age group i and family f when α is positive, and the difference between the 2.5^{th} quantile for age group i and family f and the median value for age group i and family f when α is negative. The second source is the same as the first but represents age-specific variation within the entire HMD dataset. The calculation is the same except all of the HMD life tables are included. As α approaches an absolute value of 1 and the resulting age pattern moves farther from the family-specific underlying age pattern, more weight is given to the differences calculated from the the entire dataset. When α is 1 the balance is about half-half, and when α moves beyond 1 the 'all-tables' age-specific deviation progressively dominates. Consequently, as the family-specific

level approaches the 'edge' of the data for each family, the cluster invariant (whole data set) age-specific deviation takes over. This produces smooth family-age-specific deviations for mortality levels approaching, at and beyond the levels represented by the data within each family, and as a result we are able to extrapolate reasonably to very low and high mortality levels within all of the families.

The age-specific deviations are represented by our model in the following way. Within a group of life tables (either a family or the whole HMD data set), the age-specific change $\mathbf{D}$ below or above the median is represented by:

$$\text{below the median}: \quad \mathbf{D}_{f_-} = \mathbf{M}_f\,[\text{median}] - \mathbf{M}_f\,[2.5^{\text{th}}\text{ quantile}] \tag{6}$$

$$\text{above the median}: \quad \mathbf{D}_{f_+} = \mathbf{M}_f\,[97.5^{\text{th}}\text{ quantile}] - \mathbf{M}_f\,[\text{median}] \tag{7}$$

The final expression for a life table in family f at level α is:

$$\text{when } \alpha < 0: \quad \mathbf{M}_f(\alpha) = \mathbf{M}_f + \alpha \cdot \left(\mathrm{e}^{-0.75\cdot|\alpha|} \cdot \mathbf{D}_{f_-} + (1 - \mathrm{e}^{-0.75\cdot|\alpha|}) \cdot \mathbf{D}_{h_-}\right) \tag{8}$$

$$\text{when } \alpha = 0: \quad \mathbf{M}_f(\alpha) = \mathbf{M}_f \tag{9}$$

$$\text{when } \alpha > 0: \quad \mathbf{M}_f(\alpha) = \mathbf{M}_f + \alpha \cdot \left(\mathrm{e}^{-0.75\cdot|\alpha|} \cdot \mathbf{D}_{f_+} + (1 - \mathrm{e}^{-0.75\cdot|\alpha|}) \cdot \mathbf{D}_{h_+}\right) \tag{10}$$

where f indexes families and h is the whole HMD dataset. The minus sign in (f or h)$_-$ indicates age-specific deviations below the median, and the plus sign indicates age-specific deviations above the median. The value of -0.75 for the coefficient in the exponential weight is chosen so that when $|\alpha| = 1.0$ there is approximately equal weight given to the family-specific and all-HMD age-specific deviations.

Similar to previous systems, we index the levels within each family on the expectation of life at birth. We use the `optimize()` function in the `R` statistical package (R Development Core Team, 2006) to find values of α such that the expectation of life at birth ranges from 30 to 90 within each family.

Users of the system can supply any of a variety of measures of child or adult mortality and discriminant analysis can be used to identify the family in which the supplied measure is most likely to fit.[3] Then the user can select a level or desired life expectancy within the identified family.

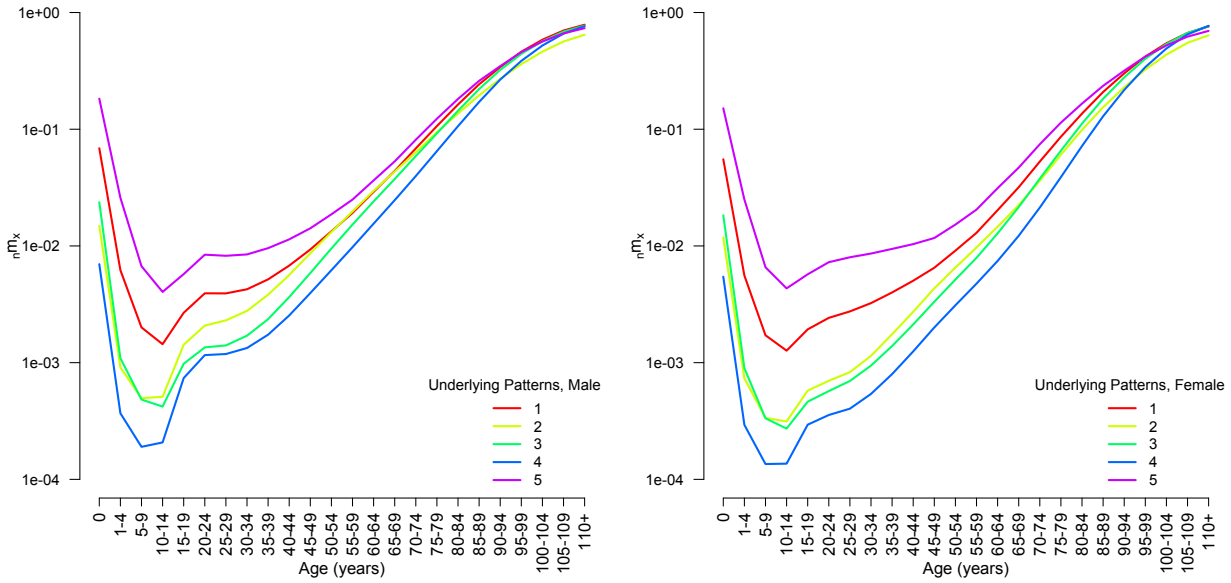
4 Results

4.1 Mortality Patterns

We identify five distinct mortality patterns for males and females. The underlying family patterns are presented in figure 4. The left panel of that figure shows that nearly all of the

[3]The `R` package `Mclust`, which was used to perform the cluster analysis, contains functions to perform discriminant analysis where any portion of the age range or any summary measure like e_0 can be used as the training data.

male patterns contain the expected young-adult mortality hump. Patterns 2 and 3 show similar levels of child mortality while diverging somewhat in adult mortality. The rest of the patterns show wide variation in both adult and child mortality for both males and females.



(a) HMD Family Patterns, male

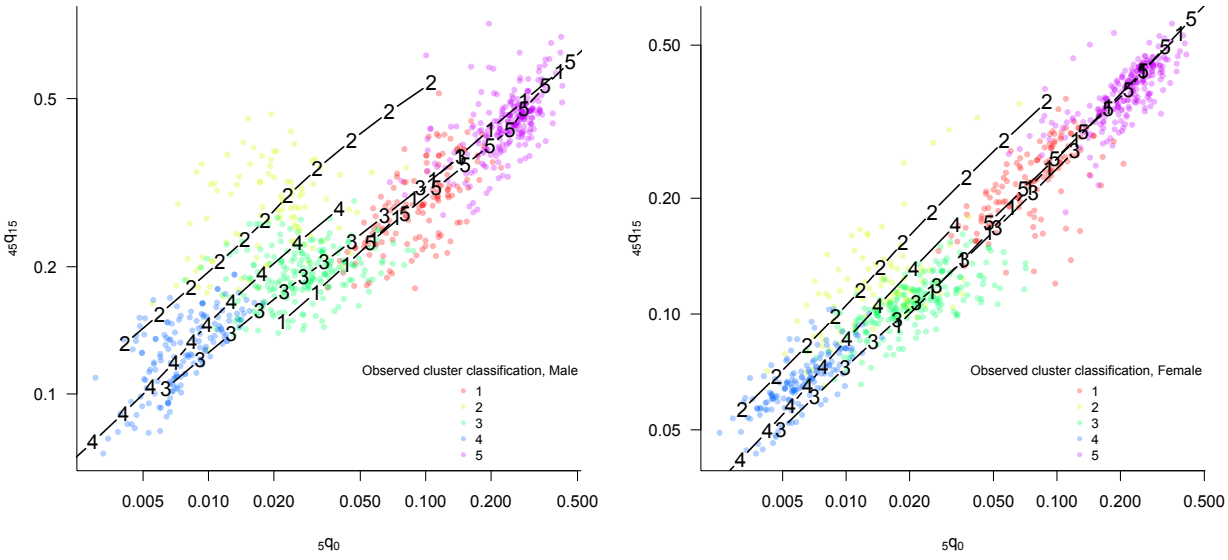
(b) HMD Family Patterns, female

Figure 4. HMD Cluster/Family Age-Patterns of Mortality (log scale)

Figure 5 plots the relationship between adult mortality (${}_{45}q_{15}$) and child mortality (${}_{5}q_{0}$) in the observed data (small circles) as well as the resulting relationships from each model family after varying α. This figure reveals that by varying α this system can cover a wide range of human mortality experiences. For example, patterns 2 and 4 cover cases of low child mortality at both high and low adult mortality respectively, while patterns 1 and 5 result from very high adult and child mortality regimes.

4.2 Components of each Cluster/Family

Each cluster or family contains life tables from a wide range of geographic locations. In contrast, the life tables in each cluster tend to come from similar historical periods. For instance patterns 2 and 3 do not contain any life tables with start dates before 1945, while cluster 4 contains no life tables with start dates prior to 1970. Cluster 1 contains life tables in a middle range with start dates between 1890 and 1960, and cluster 5 contains life tables with the earliest start dates (1770, 1774). Appendix A contains lists of cluster/family membership for all the HMD life tables sorted by time and location within time, and Appendix B contains the same lists sorted by location and time within location.



(a) male **(b)** female

Figure 5. Child versus Adult Mortality. Observed data (small colored circles) and implied relationships after varying α from -1 to 1 (black numbers correspond index families and both adult and child mortality displayed on log scale)

4.3 Family-specific Parameters

Below in Tables 8, 9, 10, 11, 12 and 13 in Appendix C are the component score vectors, median coefficients, and below-median and above-median deviations for males and females that when inserted into Equations 8 through 10 produce the model life table schedules.

4.4 Evaluating System Performance

We assess the performance of the system in several ways, starting with how well it can classify an arbitrary mortality indicator into one of the families. Because users typically approach model life table systems with at least some information on the age pattern of mortality (e.g. ${}_1m_0$, ${}_5q_0$, ${}_{45}q_{15}$), we test the use of discriminant analysis to classify partial mortality schedules. Using the known classification from the cluster analysis as a training data set, we attempt to reclassify each schedule from the HMD using one of four child mortality indicators alone and combining them with ${}_{45}q_{15}$ as a second mortality measure to determine the appropriate family. We compare the known classification from the cluster analysis of the complete schedules with the classification of the mortality indicators resulting from discriminant analysis. The percentage of HMD schedules that were misclassified are presented in table 1. For all four child mortality measures alone, roughly 25-29 percent of schedules were misclassified, while the addition of ${}_{45}q_{15}$ reduces misclassification by roughly 10-13 percentage points across all measures. When an observation is misclassified, it is typically misclassified into an adjacent cluster because when clusters are close or overlapping

classification uncertainty is greatest.

Table 1. Percent misclassified HMD schedules when using one of four child mortality measures alone and combined with ${}_{45}q_{15}$

	without ${}_{45}q_{15}$		**with** ${}_{45}q_{15}$	
	Male	Female	Male	Female
${}_1m_0$	28.8	28.1	15.2	14.8
${}_5m_0$	26.9	25.5	14.8	14.2
${}_1q_0$	28.9	28.2	14.9	15.9
${}_5q_0$	26.5	25.4	14.5	14.3

We then assess how well the model reproduces known mortality schedules. First we compare the accuracy of estimation for the system described in this paper with a recent method advanced by Wilmoth et al. (2009). Wilmoth and colleagues report that their system performs similarly to the WHO system (Murray et al., 2003) and outperforms the widely used Coale and Demeny (1966) and United Nations (1982). Following the procedure described by Wilmoth et al. (2009) we use both Wilmoth's and our systems to predict the WHO model life table database and compare the means and standard deviations of the resulting absolute errors for three mortality indicators, e_0, ${}_1m_0$ and ${}_{45}q_{15}$. Results are presented in Tables 2 and 3. The system by Wilmoth et al. (2009) allows the user to either use a single measure of child mortality (${}_5q_0$) or child mortality combined with a measure of adult mortality (${}_{45}q_{15}$) as input parameters. We present both.

Table 2. Mean absolute error (MAE) for e_0, ${}_1m_0$, ${}_{45}q_{15}$ predicting WHO life tables with HMD-calibrated system

	Female			Male		
	e_0	${}_1m_0$	${}_{45}q_{15}$	e_0	${}_1m_0$	${}_{45}q_{15}$
Clark (level: e_0)	0.010	0.009	0.010	0.007	0.006	0.015
Clark (level: ${}_{45}q_{15}$)	1.155	0.010	0.000	1.237	0.010	0.000
Wilmoth (w/o ${}_{45}q_{15}$)	1.349	0.002	0.020	1.843	0.002	0.039
Wilmoth (w/ ${}_{45}q_{15}$)	1.343	0.002	0.020	1.839	0.002	0.039

In our system we use discriminant analysis to identify the family most similar to the supplied mortality indicator, in this case mortality in the first two age groups 0 and 1-4 years at last birthday. This is accomplished using discriminant analysis that is trained to classify mortality in those two age groups into one of the families. The discriminant analysis functionality is an extension of the clustering method and built into the `R` package `Mclust` (Fraley and Raftery, 2006) that implements the model-based clustering algorithm we use to

cluster the mortality age profiles. Typically, our system (referred to as 'Clark' in the tables below), selects α (the level) to match e_0 , which is presented in the table, but for comparison we have also used ${}_{45}q_{15}$. In this latter case, we match ${}_{45}q_{15}$ without additional information about child mortality, although it would be an option to include both measures of child and adult mortality in the discriminant analysis if we want.

Table 3. Standard deviations of absolute errors for e_0, ${}_1m_0$, ${}_{45}q_{15}$ predicting WHO life tables with HMD-calibrated system

	Female			Male		
	e_0	${}_1m_0$	${}_{45}q_{15}$	e_0	${}_1m_0$	${}_{45}q_{15}$
Clark (level: e_0)	0.112	0.008	0.011	0.093	0.009	0.013
Clark (level: ${}_{45}q_{15}$)	1.180	0.013	0.001	1.182	0.016	0.001
Wilmoth (w/o ${}_{45}q_{15}$)	1.112	0.006	0.021	1.685	0.006	0.040
Wilmoth (w/ ${}_{45}q_{15}$)	1.162	0.006	0.022	1.619	0.006	0.037

The values in Tables 2 and 3 indicate that our system does well in comparison to Wilmoth and colleagues' (Wilmoth et al., 2009), which in turn does well compared to the traditional Coale and Demeny (Coale and Demeny, 1966; Coale and Guo, 1989), United Nations (United Nations, 1982) and WHO modified logit system (Murray et al., 2003). The mean absolute errors (Table 2) are substantially smaller for e_0 and ${}_{45}q_{15}$ and roughly similar for ${}_1m_0$. The latter is unsurprising because Wilmoth's model uses child mortality as the entry into the system, so in that system child mortality is matched very closely by definition. There is a similar finding for the standard deviations of the mean absolute errors (Table 3), although some of the values for our system are about equal or a little greater than Wilmoth's. Keep in mind, however, that the *mean absolute errors* are generally smaller for our system, indicating that our errors are indeed smaller on average, if as or slightly more variable.

Next, we conduct a more thorough validation by including life tables from the WHO life table database (World Health Organization, 2010). We combine the HMD and WHO life tables into a single data set and then conduct an out-of-sample validation using the combined data set. We take a random 50% sample from the combined dataset and fit this sample using both models (Wilmoth's and ours) and then predict the remaining half of the life tables using the model 'fits' - i.e. both Wilmoth's and our models calibrated to the 50% random sample. To select the appropriate family in our system we use discriminant analysis with the first two age groups (0, 1-4) as the training set and then select the level by finding the value of α necessary to produce a life table whose life expectancy matches as closely as possible the observed life expectancy in the out-of-sample life table. We then calculate the means and standard deviations of the absolute errors for three mortality indicators e_0, ${}_1m_0$ and ${}_{45}q_{15}$, displayed in Tables 4 and 5.

Table 4. Mean absolute error (MAE) for e_0, ${}_1m_0$, ${}_{45}q_{15}$ predicting out-of-sample 50% HMD/WHO combined life tables

	Female			Male		
	e_0	${}_1m_0$	${}_{45}q_{15}$	e_0	${}_1m_0$	${}_{45}q_{15}$
Clark (level: e_0)	0.007	0.006	0.011	0.027	0.007	0.015
Clark (level: ${}_{45}q_{15}$)	1.361	0.009	0.000	1.407	0.013	0.000
Wilmoth (w/o ${}_{45}q_{15}$)	1.337	0.003	0.021	1.892	0.003	0.041
Wilmoth (w/ ${}_{45}q_{15}$)	1.311	0.003	0.021	1.874	0.003	0.041

Both models were re-estimated with 50% in-sample half of HMD/WHO combined data set

Table 5. Standard deviations of absolute errors for e_0, ${}_1m_0$, ${}_{45}q_{15}$ predicting out-of-sample 50% HMD/WHO combined life tables

	Female			Male		
	e_0	${}_1m_0$	${}_{45}q_{15}$	e_0	${}_1m_0$	${}_{45}q_{15}$
Clark (level: e_0)	0.099	0.009	0.013	0.243	0.011	0.017
Clark (level: ${}_{45}q_{15}$)	1.478	0.017	0.000	1.722	0.027	0.000
Wilmoth (w/o ${}_{45}q_{15}$)	1.100	0.007	0.024	1.695	0.008	0.040
Wilmoth (w/ ${}_{45}q_{15}$)	1.112	0.007	0.024	1.574	0.008	0.036

Both models were re-estimated with 50% in-sample half of HMD/WHO combined data set

The results for this additional validation are very similar to the first, but more fair. In the previous test we replicated the approach of Wilmoth and colleagues so that we could compare our results directly with theirs, i.e. calibrating with HMD life tables and predicting WHO life tables. In our second test we calibrated our system using a random half of the same dataset we are predicting, thus ensuring that the data used to fit the models fundamentally has the same structure (describes the same universe of mortality) as the data used to assess the predictions.

5 Discussion

We present a parsimonious, robust model of age-specific mortality and a method for identifying common age patterns of mortality. Together these allow us to create a traditional system of model life tables that includes a small number of families with varying levels of mortality within each family. We 'calibrate' this model using the life tables in the HMD life table database to produce a model life table system that describes the variation in age-patterns of mortality in the HMD life tables. We present various measures of goodness of fit and

conduct an out-of-sample validation using life tables in both the HMD and WHO life table databases. Our new model life table system performs well compared to the recent model proposed by Wilmoth et al. (2009) which itself generally outperforms the other commonly used existing model life table systems. Our new families include some that combine low child mortality and relatively high adult mortality that do not appear in any existing model life table systems – i.e. families 2 and 3.

Missing from our calibrated model is mortality experience in the developing world or in populations with unusual demographic or epidemiological circumstances, e.g. war-time mortality or high HIV prevalence. We are addressing this limitation through a collaboration with the INDEPTH network (INDEPTH Network, 2011) of health and demographic surveillance system sites in Africa and Asia. INDEPTH has compiled a large collection of life tables from its member sites, and we are applying our model life table framework to the INDEPTH collection both with and without the HMD and WHO life tables. Many of the INDEPTH life tables describe mortality experience in populations with unusual demographic and epidemiological experiences – including very high HIV prevalence.

Our model life table system maintains the attractive features of the traditional model life table systems – i.e. parsimony, intuitive interpretation and ease of use – while introducing a new fully automated and statistically valid method of construction. This removes any 'analyst bias' from the process of identifying the empirical regularities in a collection of life tables and produces reproducible classifications of life tables to families. Our model of age-specific mortality allows easy, flexible manipulation of mortality age-patterns which enables us to construct a simple model of mortality age-patterns within each family. As demonstrated by the out-of-sample validation, the overall modeling framework works well with different collections of model life tables, which will allow us to rapidly incorporate the variation in age-patterns of mortality described by the INDEPTH life tables. Perhaps most important, this framework (along with the tool described immediately below) is easy to use and relatively flexible. The user can approach the model life table system with a variety of summary mortality indicators with which they can identify the most similar mortality family and appropriate level within that family. The discriminant analysis functionality is flexible and allows the user to train the system using a combination of mortality indicators that is not included in the basic release of our tool (below). Finally, this general approach (essentially a linear system) easily lends itself to many interesting and useful additions such as interpolating between two or more families and adding additional information into the clustering stage of the analysis, such as direct measures of HIV prevalence and age of the HIV epidemic for populations with high levels of HIV prevalence.

To make this model life table system accessible and useful, we have created a package for the free, open source statistical package `R` (R Development Core Team, 2006) with a user-friendly graphical interface that implements the HMD-calibrated version of our model. The package will be available from the `CRAN` archive soon and can be requested from the authors immediately.

Finally, all of the information necessary to generate life tables in our HMD-calibrated system is available in the appendices.

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References

Brass, W. 1971. "On the Scale of Mortality." In *Biological Aspects of Demography*, edited by W. Brass, London: Taylor and Francis.

Clark, S. J. 2002. "INDEPTH Mortality Patterns for Africa." In *Population and Health in Developing Countries*, vol. 1, edited by INDEPTH network, Ottawa: IDRC Press.

Clark, S. J., M. Jasseh, S. Punpuing, E. Zulu, O. S. Ayaga Bawah and INDEPTH Network Member Sites. 2009. "INDEPTH Model Life Tables 2.0." Presented at the Annual Meeting for the Population Association of America.

Coale, A. J. and P. Demeny. 1966. *Regional Model Life Tables and Stable Populations*. Princeton University Press.

Coale, A. J. and G. Guo. 1989. "Revised regional model life tables at very low levels of mortality." *Population Index* 55(4):613–643.

Fraley, C. and A. Raftery. 2006. "Mclust version 3 for R: Normal mixture modeling and model-based clustering." Tech. Rep. 504, Department of Statistics, University of Washington.

Gompertz, B. 1825. "On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies." *Philosophical Transactions of the Royal Society of London* 115:513–583.

Heligman, L. and J. H. Pollard. 1980. "The Age Pattern of Mortality." *Journal of the Institute of Actuaries* 107(434):49–80.

INDEPTH Network. 2011. "INDEPTH Network – International Network for the Demographic Evaluation of Populations and Their Health in Developing Countries." URL `http://www.indepth-network.org`.

Murray, C., B. Ferguson, A. Lopez, M. Guillot, J. Salomon and O. Ahmad. 2003. "Modified Logit Life Table System: Principles, Empirical Validation, and Application." *Population Studies* 57:165–182.

R Development Core Team. 2006. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. URL `http://www.R-project.org`, ISBN 3-900051-07-0.

United Nations. 1982. "Model Life Tables for Developing Countries." In *Population Studies*, 77, New York: United Nations.

University of California, Berkeley and Max Planck Institute for Demographic Research. 2009. "Human Mortality Database." URL `http://www.mortality.org`.

Wilmoth, J., K. Andreev, D. Jdanov and D. Glei. 2007. "Methods Protocol for the Human Mortality Database Version 5." www.mortality.org/Public/Docs/MethodsProtocol.pdf.

Wilmoth, J., V. Canudas-Romo, S. Zureick, M. Inoue and C. Sawyer. 2009. "A Flexible Two-Dimensional Mortality Model for Use in Indirect Estimation." Presented at the Annual Meeting for the Population Association of America.

World Health Organization. 2010. "WHO Mortality Database." URL `http://www.who.int/healthinfo/morttables/en/index.html`.

Appendix A - Cluster membership by period then location within period

Table 6. Classified Period and Countries

Period	Country/Region
	Family 1
1890-1894	New Zealand: Non-Maori Population
1895-1899	New Zealand: Non-Maori Population
1900-1904	New Zealand: Non-Maori Population
1905-1909	New Zealand: Non-Maori Population
1910-1914	Denmark, England & Wales, Iceland, New Zealand: Non-Maori Population, Sweden, Switzerland
1915-1919	Denmark, England & Wales, New Zealand: Non-Maori Population, Scotland, Switzerland
1920-1924	Belgium, Denmark, England & Wales, France, Netherlands, New Zealand: Non-Maori Population, Scotland, Sweden, Switzerland
1921-1924	Australia, Canada
1922-1924	Northern Ireland
1925-1929	Australia, Belgium, Canada, Denmark, England & Wales, France, Netherlands, New Zealand: Non-Maori Population, Norway, Scotland, Sweden, Switzerland
1930-1934	Australia, Belgium, Canada, Denmark, England & Wales, Finland, France, Netherlands, New Zealand: Non-Maori Population, Northern Ireland, Norway, Scotland, Spain, Sweden, Switzerland
1933-1934	USA
1935-1939	Australia, Belgium, Canada, Denmark, England & Wales, Finland, France, Iceland, Netherlands, New Zealand: Non-Maori Population, Northern Ireland, Norway, Scotland, Sweden, Switzerland, USA
1940-1944	Australia, Belgium, Canada, Denmark, England & Wales, France, Iceland, Italy, Netherlands, New Zealand: Non-Maori Population, Northern Ireland, Norway, Portugal, Scotland, Spain, Sweden, Switzerland, USA
1945-1949	Belgium, Canada, Denmark, England & Wales, Finland, France, Iceland, Netherlands, Northern Ireland, Norway, Portugal, Scotland, Spain, Sweden, Switzerland
1947-1949	Austria, Japan
1948-1949	New Zealand: National Population
1950-1954	Bulgaria, Hungary, Ireland, Japan, Portugal, Slovak Republic, Spain
1955-1959	Bulgaria, Japan, Portugal, Spain
1960-1964	Portugal

Continued on Next Page...

Table 6 – Continued

Period	Country/Region
	Family 2
1945-1949	USA
1950-1954	USA
1955-1959	New Zealand: Maori Population
1960-1964	Belarus, Latvia, Lithuania, New Zealand: Maori Population, Russia, Ukraine
1965-1969	Belarus, Estonia, Latvia, Lithuania, New Zealand: Maori Population, Russia, Ukraine, USA
1970-1974	Belarus, Estonia, Latvia, Lithuania, New Zealand: Maori Population, Russia, Ukraine, USA
1975-1979	Belarus, Canada, Estonia, Finland, Latvia, Lithuania, New Zealand: Maori Population, Russia, Taiwan, Ukraine, USA
1980-1984	Belarus, Estonia, Finland, Hungary, Japan, Latvia, Lithuania, New Zealand: Maori Population, Russia, Taiwan, Ukraine, USA
1985-1989	Belarus, Estonia, Finland, Hungary, Latvia, Lithuania, New Zealand: Maori Population, Russia, Taiwan, Ukraine, USA
1990-1994	Belarus, Estonia, Germany: East, Hungary, Latvia, Lithuania, New Zealand: Maori Population, Poland, Russia, Slovenia, Taiwan, Ukraine, USA
1995-1999	Belarus, Estonia, Hungary, Latvia, Lithuania, New Zealand: Maori Population, Poland, Russia, Slovenia, Taiwan, Ukraine
2000-2003	New Zealand: Maori Population
2000-2004	Belarus, Estonia, Hungary, Latvia, Lithuania, Poland, Russia, Slovenia, Ukraine
2005-2006	Hungary, Poland, Russia, Ukraine
2005-2007	Belarus, Estonia, Latvia, Lithuania
	Family 3
1945-1949	Australia, New Zealand: Non-Maori Population
1950-1954	Australia, Austria, Belgium, Canada, Czech Republic, Denmark, England & Wales, Finland, France, Iceland, Italy, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Northern Ireland, Norway, Scotland, Sweden, Switzerland
1955-1959	Australia, Austria, Belgium, Canada, Czech Republic, Denmark, England & Wales, Finland, France, Hungary, Iceland, Ireland, Italy, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Northern Ireland, Norway, Slovak Republic, Scotland, Sweden, Switzerland, USA
1956-1959	Germany: East, Germany: West
1958-1959	Poland

Continued on Next Page...

Table 6 – Continued

Period	Country/Region
1960-1964	Australia, Austria, Belgium, Bulgaria, Canada, Czech Republic, Denmark, England & Wales, Estonia, Finland, France, Germany: East, Germany: West, Hungary, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Northern Ireland, Norway, Poland, Slovak Republic, Scotland, Spain, Sweden, Switzerland, USA
1965-1969	Australia, Austria, Belgium, Bulgaria, Canada, Czech Republic, Denmark, England & Wales, Finland, France, Germany: East, Germany: West, Hungary, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Northern Ireland, Norway, Poland, Portugal, Slovak Republic, Scotland, Spain, Sweden, Switzerland
1970-1974	Australia, Austria, Belgium, Bulgaria, Canada, Czech Republic, Denmark, England & Wales, Finland, France, Germany: East, Germany: West, Hungary, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Northern Ireland, Norway, Poland, Portugal, Slovak Republic, Scotland, Spain, Switzerland, Taiwan
1975-1979	Australia, Austria, Belgium, Bulgaria, Czech Republic, Denmark, England & Wales, France, Germany: East, Germany: West, Hungary, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Northern Ireland, Norway, Poland, Portugal, Slovak Republic, Scotland, Spain
1980-1984	Austria, Bulgaria, Czech Republic, England & Wales, Germany: East, Germany: West, Ireland, Italy, Luxembourg, Northern Ireland, Norway, Poland, Portugal, Slovak Republic, Scotland, Spain
1983-1984	Slovenia
1985-1989	Austria, Bulgaria, Czech Republic, Germany: East, Poland, Portugal, Slovak Republic, Scotland, Slovenia
1990-1994	Bulgaria, Czech Republic, Slovak Republic
1995-1999	Bulgaria, Czech Republic, Slovak Republic
2000-2004	Bulgaria, Slovak Republic
2005-2006	Slovak Republic
2005-2007	Bulgaria
	Family 4
1970-1974	Sweden
1975-1979	Sweden, Switzerland

Continued on Next Page...

Table 6 – Continued

Period	Country/Region
1980-1984	Australia, Belgium, Canada, Denmark, France, Iceland, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Sweden, Switzerland
1985-1989	Australia, Belgium, Canada, Denmark, England & Wales, France, Germany: West, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Northern Ireland, Norway, Spain, Sweden, Switzerland
1990-1994	Australia, Austria, Belgium, Canada, Denmark, England & Wales, Finland, France, Germany: West, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Northern Ireland, Norway, Portugal, Scotland, Spain, Sweden, Switzerland
1991-1994	Germany
1992-1994	Chile
1995-1999	Australia, Austria, Belgium, Canada, Chile, Denmark, England & Wales, Finland, France, Germany, Germany: East, Germany: West, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand: Non-Maori Population, New Zealand: National Population, Northern Ireland, Norway, Portugal, Scotland, Spain, Sweden, Switzerland, USA
2000-2003	New Zealand: Non-Maori Population, New Zealand: National Population
2000-2004	Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, England & Wales, Finland, France, Germany, Germany: East, Germany: West, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, Northern Ireland, Norway, Portugal, Scotland, Spain, Sweden, Switzerland, Taiwan, USA
2005-2006	Belgium, Czech Republic, England & Wales, France, Germany, Germany: East, Germany: West, Ireland, Italy, Luxembourg, Netherlands, Northern Ireland, Scotland, Slovenia, Spain
2005-2007	Denmark, Finland, Iceland, Japan, Norway, Portugal, Sweden, Switzerland, Taiwan
	Family 5
1751-1754	Sweden
1755-1759	Sweden
1760-1764	Sweden
1765-1769	Sweden
1770-1774	Sweden
1775-1779	Sweden
1780-1784	Sweden

Continued on Next Page...

Table 6 – Continued

Period	Country/Region
1785-1789	Sweden
1790-1794	Sweden
1795-1799	Sweden
1800-1804	Sweden
1805-1809	Sweden
1810-1814	Sweden
1815-1819	Sweden
1816-1819	France
1820-1824	France, Sweden
1825-1829	France, Sweden
1830-1834	France, Sweden
1835-1839	Denmark, France, Sweden
1838-1839	Iceland
1840-1844	Denmark, France, Iceland, Sweden
1841-1844	Belgium, England & Wales
1845-1849	Belgium, Denmark, England & Wales, France, Iceland, Sweden
1846-1849	Norway
1850-1854	Belgium, Denmark, England & Wales, France, Iceland, Netherlands, Norway, Sweden
1855-1859	Belgium, Denmark, England & Wales, France, Iceland, Netherlands, Norway, Scotland, Sweden
1860-1864	Belgium, Denmark, England & Wales, France, Iceland, Netherlands, Norway, Scotland, Sweden
1865-1869	Belgium, Denmark, England & Wales, France, Iceland, Netherlands, Norway, Scotland, Sweden
1870-1874	Belgium, Denmark, England & Wales, France, Iceland, Netherlands, Norway, Scotland, Sweden
1872-1874	Italy
1875-1879	Belgium, Denmark, England & Wales, France, Iceland, Italy, Netherlands, Norway, Scotland, Sweden
1876-1879	New Zealand: Non-Maori Population, Switzerland
1878-1879	Finland
1880-1884	Belgium, Denmark, England & Wales, Finland, France, Iceland, Italy, Netherlands, New Zealand: Non-Maori Population, Norway, Scotland, Sweden, Switzerland
1885-1889	Belgium, Denmark, England & Wales, Finland, France, Iceland, Italy, Netherlands, New Zealand: Non-Maori Population, Norway, Scotland, Sweden, Switzerland
1890-1894	Belgium, Denmark, England & Wales, Finland, France, Iceland, Italy, Netherlands, Norway, Scotland, Sweden, Switzerland

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Table 6 – Continued

Period	Country/Region
1895-1899	Belgium, Denmark, England & Wales, Finland, France, Iceland, Italy, Netherlands, Norway, Scotland, Sweden, Switzerland
1900-1904	Belgium, Denmark, England & Wales, Finland, France, Iceland, Italy, Netherlands, Norway, Scotland, Sweden, Switzerland
1905-1909	Belgium, Denmark, England & Wales, Finland, France, Iceland, Italy, Netherlands, Norway, Scotland, Sweden, Switzerland
1908-1909	Spain
1910-1914	Belgium, Finland, France, Italy, Netherlands, Norway, Scotland, Spain
1915-1919	Belgium, England & Wales, Finland, France, Iceland, Italy, Netherlands, Norway, Spain, Sweden
1920-1924	Finland, Iceland, Italy, Norway, Spain
1925-1929	Finland, Iceland, Italy, Northern Ireland, Spain
1930-1934	Iceland, Italy
1935-1939	Italy, Spain
1937-1939	New Zealand: Maori Population
1940-1944	Finland, New Zealand: Maori Population
1945-1949	Italy, New Zealand: Maori Population
1947-1949	Bulgaria
1950-1954	New Zealand: Maori Population

Appendix B - Cluster membership by location then period within location

Table 7. Classified Countries and Periods

Country/Region	Period
	Family 1
Australia	1921-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944
Austria	1947-1949
Belgium	1920-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
Bulgaria	1950-1954, 1955-1959
Canada	1921-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
Denmark	1910-1914, 1915-1919, 1920-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
England & Wales	1910-1914, 1915-1919, 1920-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
Finland	1930-1934, 1935-1939, 1945-1949
France	1920-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
Hungary	1950-1954
Iceland	1910-1914, 1935-1939, 1940-1944, 1945-1949
Ireland	1950-1954
Italy	1940-1944
Japan	1947-1949, 1950-1954, 1955-1959
Netherlands	1920-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
New Zealand: National Population	1948-1949
New Zealand: Non-Maori Population	1890-1894, 1895-1899, 1900-1904, 1905-1909, 1910-1914, 1915-1919, 1920-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944
Northern Ireland	1922-1924, 1930-1934, 1935-1939, 1940-1944, 1945-1949
Norway	1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
Portugal	1940-1944, 1945-1949, 1950-1954, 1955-1959, 1960-1964
Scotland	1915-1919, 1920-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
Slovak Republic	1950-1954

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Table 7 – Continued

Country/Region	Period
Spain	1930-1934, 1940-1944, 1945-1949, 1950-1954, 1955-1959
Sweden	1910-1914, 1920-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
Switzerland	1910-1914, 1915-1919, 1920-1924, 1925-1929, 1930-1934, 1935-1939, 1940-1944, 1945-1949
USA	1933-1934, 1935-1939, 1940-1944

	Family 2
Belarus	1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
Canada	1975-1979
Estonia	1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
Finland	1975-1979, 1980-1984, 1985-1989
Germany: East	1990-1994
Hungary	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Japan	1980-1984
Latvia	1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
Lithuania	1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
New Zealand: Maori Population	1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2003
Poland	1990-1994, 1995-1999, 2000-2004, 2005-2006
Russia	1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Slovenia	1990-1994, 1995-1999, 2000-2004
Taiwan	1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999
Ukraine	1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
USA	1945-1949, 1950-1954, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994

Continued on Next Page...

Table 7 – Continued

Country/Region	Period
	Family 3
Australia	1945-1949, 1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979
Austria	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989
Belgium	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979
Bulgaria	1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
Canada	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974
Czech Republic	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999
Denmark	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979
England & Wales	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984
Estonia	1960-1964
Finland	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974
France	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979
Germany: East	1956-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989
Germany: West	1956-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984
Hungary	1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979
Iceland	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979
Ireland	1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984
Italy	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984
Japan	1960-1964, 1965-1969, 1970-1974, 1975-1979
Luxembourg	1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984

Continued on Next Page...

Table 7 – Continued

Country/Region	Period
Netherlands	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979
New Zealand: National Population	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979
New Zealand: Non-Maori Population	1945-1949, 1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979
Northern Ireland	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984
Norway	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984
Poland	1958-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989
Portugal	1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989
Scotland	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989
Slovak Republic	1955-1959, 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Slovenia	1983-1984, 1985-1989
Spain	1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984
Sweden	1950-1954, 1955-1959, 1960-1964, 1965-1969
Switzerland	1950-1954, 1955-1959, 1960-1964, 1965-1969, 1970-1974
Taiwan	1970-1974
USA	1955-1959, 1960-1964

	Family 4
Australia	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004
Austria	1990-1994, 1995-1999, 2000-2004
Belgium	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Canada	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004
Chile	1992-1994, 1995-1999, 2000-2004
Czech Republic	2000-2004, 2005-2006
Denmark	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007

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Table 7 – Continued

Country/Region	Period
England & Wales	1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Finland	1990-1994, 1995-1999, 2000-2004, 2005-2007
France	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Germany	1991-1994, 1995-1999, 2000-2004, 2005-2006
Germany: East	1995-1999, 2000-2004, 2005-2006
Germany: West	1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Iceland	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
Ireland	1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Italy	1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Japan	1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
Luxembourg	1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Netherlands	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
New Zealand: National Population	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2003
New Zealand: Non-Maori Population	1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2003
Northern Ireland	1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Norway	1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
Portugal	1990-1994, 1995-1999, 2000-2004, 2005-2007
Scotland	1990-1994, 1995-1999, 2000-2004, 2005-2006
Slovenia	2005-2006
Spain	1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2006
Sweden	1970-1974, 1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
Switzerland	1975-1979, 1980-1984, 1985-1989, 1990-1994, 1995-1999, 2000-2004, 2005-2007
Taiwan	2000-2004, 2005-2007
USA	1995-1999, 2000-2004

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Table 7 – Continued

Country/Region	Period
	Family 5
Belgium	1841-1844, 1845-1849, 1850-1854, 1855-1859, 1860-1864, 1865-1869, 1870-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909, 1910-1914, 1915-1919
Bulgaria	1947-1949
Denmark	1835-1839, 1840-1844, 1845-1849, 1850-1854, 1855-1859, 1860-1864, 1865-1869, 1870-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909
England & Wales	1841-1844, 1845-1849, 1850-1854, 1855-1859, 1860-1864, 1865-1869, 1870-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909
Finland	1878-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909, 1910-1914, 1915-1919, 1920-1924, 1925-1929, 1940-1944
France	1816-1819, 1820-1824, 1825-1829, 1830-1834, 1835-1839, 1840-1844, 1845-1849, 1850-1854, 1855-1859, 1860-1864, 1865-1869, 1870-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909, 1910-1914, 1915-1919
Iceland	1838-1839, 1840-1844, 1845-1849, 1850-1854, 1855-1859, 1860-1864, 1865-1869, 1870-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909, 1915-1919, 1920-1924, 1925-1929, 1930-1934
Italy	1872-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909, 1910-1914, 1915-1919, 1920-1924, 1925-1929, 1930-1934, 1935-1939, 1945-1949
Netherlands	1850-1854, 1855-1859, 1860-1864, 1865-1869, 1870-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909, 1910-1914, 1915-1919
New Zealand: Maori Population	1937-1939, 1940-1944, 1945-1949, 1950-1954
New Zealand: Non-Maori Population	1876-1879, 1880-1884, 1885-1889
Northern Ireland	1925-1929

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Table 7 – Continued

Country/Region	Period
Norway	1846-1849, 1850-1854, 1855-1859, 1860-1864, 1865-1869, 1870-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909, 1910-1914, 1915-1919, 1920-1924
Scotland	1855-1859, 1860-1864, 1865-1869, 1870-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909, 1910-1914
Spain	1908-1909, 1910-1914, 1915-1919, 1920-1924, 1925-1929, 1935-1939
Sweden	1751-1754, 1755-1759, 1760-1764, 1765-1769, 1770-1774, 1775-1779, 1780-1784, 1785-1789, 1790-1794, 1795-1799, 1800-1804, 1805-1809, 1810-1814, 1815-1819, 1820-1824, 1825-1829, 1830-1834, 1835-1839, 1840-1844, 1845-1849, 1850-1854, 1855-1859, 1860-1864, 1865-1869, 1870-1874, 1875-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909, 1915-1919
Switzerland	1876-1879, 1880-1884, 1885-1889, 1890-1894, 1895-1899, 1900-1904, 1905-1909

Appendix C - Fixed model parameters

Table 8. Component score vector values: **S**

	Male				Female			
Age	v1	v2	v3	v4	v1	v2	v3	v4
0	-0.10942	-0.36456	0.31976	0.12879	-0.11662	-0.35830	0.30010	0.12385
1-4	-0.19268	-0.37429	0.03399	0.05867	-0.19766	-0.40410	0.02355	0.04919
5-9	-0.22114	-0.13449	0.02214	0.00020	-0.22922	-0.19411	-0.05173	-0.05698
10-14	-0.22648	0.00536	-0.03961	-0.04350	-0.23536	-0.09290	-0.12990	-0.12124
15-19	-0.20065	0.11199	-0.16963	-0.10127	-0.21898	-0.04539	-0.20526	-0.17254
20-24	-0.18876	0.08125	-0.23927	-0.08222	-0.21238	-0.07447	-0.19725	-0.14148
25-29	-0.18802	0.09426	-0.22089	0.01015	-0.20783	-0.07266	-0.14051	-0.09933
30-34	-0.18420	0.11459	-0.15548	0.07039	-0.20056	-0.03784	-0.09478	-0.05204
35-39	-0.17647	0.13395	-0.08794	0.13676	-0.19109	0.00947	-0.04720	0.01364
40-44	-0.16598	0.15950	-0.01295	0.15460	-0.18068	0.06429	-0.00251	0.03775
45-49	-0.15401	0.18246	0.04951	0.16376	-0.16953	0.11633	0.04152	0.06177
50-54	-0.14137	0.19088	0.10486	0.15206	-0.15761	0.12349	0.07394	0.06957
55-59	-0.12877	0.19452	0.14800	0.13230	-0.14562	0.12054	0.10188	0.07713
60-64	-0.11538	0.17457	0.16354	0.12220	-0.13168	0.08816	0.13586	0.08019
65-69	-0.10226	0.15357	0.16968	0.08376	-0.11710	0.06797	0.17252	0.04367
70-74	-0.08846	0.12047	0.16837	0.04343	-0.10088	0.04237	0.19804	0.01891
75-79	-0.07464	0.09262	0.15874	0.00487	-0.08460	0.03105	0.20083	-0.00184
80-84	-0.06118	0.07059	0.14972	-0.07238	-0.06893	0.03329	0.19223	-0.06298
85-89	-0.04834	0.05471	0.14185	-0.14277	-0.05434	0.03620	0.17129	-0.13905
90-94	-0.03706	0.04997	0.12906	-0.21531	-0.04167	0.04102	0.14671	-0.20837
95-99	-0.02727	0.03711	0.11264	-0.24883	-0.03042	0.04044	0.12333	-0.25367
100-104	-0.01941	0.03115	0.09599	-0.26567	-0.02138	0.04149	0.09961	-0.27718
105-109	-0.01337	0.02670	0.07979	-0.26262	-0.01446	0.04043	0.07889	-0.27997
110+	-0.00954	0.02356	0.06866	-0.25294	-0.01012	0.03806	0.06600	-0.27463

Table 9. Median coefficients for first 4 score vectors, by cluster/family

Cluster	Intercept	Coeff 1	Coeff 2	Coeff 3	Coeff 4
1	0.0500	29.0949	-1.2763	-0.0014	-0.0681
2	0.0269	33.5397	1.5962	-0.2199	0.6667
3	-0.0381	34.0455	0.6514	0.8543	-0.1176
4	0.0714	37.9729	1.6700	-0.7928	-0.1456
5	-0.0052	24.2750	-2.7641	-0.1395	-0.0118

Table 10. Male above-median family-specific deviations and cluster-invariant deviations: $\mathbf{D}_{f_+}$

	Family					
Age	1	2	3	4	5	$\mathbf{D}_{h_+}$
0	0.737	1.015	0.912	0.572	0.558	2.277
1-4	1.172	1.181	1.051	0.684	0.828	3.178
5-9	0.754	0.913	0.737	0.680	0.545	2.590
10-14	0.620	0.772	0.555	0.626	0.485	2.207
15-19	0.683	0.662	0.400	0.473	0.603	1.746
20-24	0.901	0.754	0.465	0.425	0.803	1.835
25-29	0.955	0.860	0.504	0.431	0.916	1.837
30-34	0.868	0.880	0.494	0.443	0.889	1.722
35-39	0.788	0.905	0.484	0.446	0.876	1.583
40-44	0.630	0.849	0.428	0.440	0.769	1.359
45-49	0.492	0.790	0.373	0.425	0.675	1.139
50-54	0.365	0.719	0.324	0.409	0.570	0.951
55-59	0.260	0.649	0.279	0.390	0.478	0.786
60-64	0.258	0.632	0.280	0.366	0.469	0.725
65-69	0.242	0.582	0.266	0.337	0.430	0.657
70-74	0.257	0.545	0.269	0.308	0.411	0.629
75-79	0.284	0.511	0.269	0.274	0.407	0.594
80-84	0.255	0.417	0.235	0.237	0.333	0.513
85-89	0.222	0.327	0.198	0.201	0.264	0.421
90-94	0.179	0.228	0.148	0.163	0.189	0.309
95-99	0.200	0.196	0.138	0.133	0.189	0.266
100-104	0.229	0.183	0.133	0.106	0.209	0.228
105-109	0.278	0.197	0.141	0.086	0.258	0.214
110+	0.319	0.218	0.151	0.073	0.303	0.213

Table 11. Male below-median family-specific deviations and cluster-invariant deviations: $\mathbf{D}_{f\_}$

	Family					
Age	1	2	3	4	5	$\mathbf{D}_{h\_}$
0	0.731	0.902	0.911	0.777	1.069	1.934
1-4	1.061	1.168	0.986	0.868	1.187	2.245
5-9	0.791	0.960	0.688	0.709	0.844	1.779
10-14	0.697	0.859	0.520	0.614	0.691	1.466
15-19	0.717	0.799	0.393	0.521	0.661	1.110
20-24	0.850	0.858	0.456	0.562	0.755	1.157
25-29	0.874	0.832	0.489	0.620	0.707	1.192
30-34	0.809	0.769	0.481	0.635	0.630	1.169
35-39	0.746	0.700	0.477	0.651	0.555	1.139
40-44	0.636	0.619	0.433	0.623	0.481	1.048
45-49	0.538	0.546	0.391	0.591	0.421	0.952
50-54	0.451	0.492	0.355	0.553	0.391	0.871
55-59	0.379	0.450	0.323	0.514	0.377	0.797
60-64	0.372	0.447	0.332	0.503	0.411	0.782
65-69	0.361	0.452	0.328	0.474	0.458	0.751
70-74	0.370	0.472	0.339	0.451	0.528	0.744
75-79	0.386	0.492	0.347	0.429	0.595	0.727
80-84	0.369	0.506	0.325	0.374	0.661	0.670
85-89	0.349	0.514	0.301	0.321	0.716	0.607
90-94	0.322	0.515	0.264	0.262	0.758	0.524
95-99	0.334	0.527	0.261	0.240	0.803	0.494
100-104	0.351	0.535	0.261	0.228	0.833	0.468
105-109	0.378	0.543	0.271	0.232	0.854	0.460
110+	0.402	0.548	0.282	0.242	0.868	0.461

Table 12. Female above-median family-specific deviations and cluster-invariant deviations: $\mathbf{D}_{f_+}$

	Family					
Age	1	2	3	4	5	$\mathbf{D}_{h_+}$
0	0.755	1.021	0.914	0.581	0.569	2.328
1-4	1.221	1.205	1.091	0.702	0.843	3.316
5-9	0.915	0.942	0.806	0.688	0.611	2.875
10-14	0.845	0.818	0.666	0.641	0.571	2.624
15-19	0.883	0.740	0.581	0.552	0.621	2.379
20-24	0.951	0.800	0.631	0.548	0.689	2.437
25-29	0.888	0.816	0.633	0.559	0.661	2.372
30-34	0.807	0.815	0.599	0.549	0.646	2.193
35-39	0.728	0.827	0.558	0.531	0.653	1.969
40-44	0.596	0.773	0.480	0.505	0.593	1.685
45-49	0.469	0.722	0.405	0.478	0.537	1.406
50-54	0.413	0.696	0.379	0.457	0.506	1.265
55-59	0.381	0.681	0.367	0.437	0.490	1.160
60-64	0.375	0.681	0.386	0.422	0.478	1.126
65-69	0.300	0.611	0.361	0.401	0.389	1.017
70-74	0.270	0.570	0.353	0.374	0.346	0.931
75-79	0.271	0.536	0.337	0.334	0.342	0.824
80-84	0.227	0.441	0.281	0.285	0.282	0.659
85-89	0.188	0.334	0.219	0.233	0.217	0.502
90-94	0.160	0.238	0.163	0.185	0.166	0.361
95-99	0.165	0.181	0.130	0.144	0.154	0.266
100-104	0.192	0.156	0.112	0.109	0.173	0.198
105-109	0.238	0.162	0.113	0.083	0.220	0.166
110+	0.278	0.177	0.120	0.068	0.261	0.157

Table 13. Female below-median Family-Specific deviations and cluster-invariant deviations: $\mathbf{D}_{f\_}$

	Family					
Age	1	2	3	4	5	$\mathbf{D}_{h\_}$
0	0.746	0.914	0.908	0.780	1.066	1.945
1-4	1.099	1.210	1.020	0.884	1.232	2.323
5-9	0.909	1.073	0.746	0.725	0.976	1.907
10-14	0.863	1.023	0.616	0.642	0.890	1.669
15-19	0.877	1.001	0.548	0.581	0.898	1.476
20-24	0.917	1.019	0.595	0.614	0.934	1.538
25-29	0.870	0.978	0.600	0.628	0.892	1.541
30-34	0.807	0.908	0.573	0.626	0.813	1.467
35-39	0.741	0.821	0.540	0.630	0.714	1.370
40-44	0.643	0.727	0.476	0.597	0.616	1.224
45-49	0.549	0.636	0.414	0.565	0.524	1.080
50-54	0.505	0.597	0.397	0.549	0.504	1.022
55-59	0.477	0.571	0.394	0.540	0.502	0.987
60-64	0.469	0.570	0.419	0.541	0.540	1.005
65-69	0.419	0.554	0.406	0.504	0.569	0.964
70-74	0.396	0.548	0.409	0.480	0.610	0.940
75-79	0.390	0.541	0.405	0.457	0.646	0.889
80-84	0.359	0.528	0.364	0.398	0.677	0.783
85-89	0.332	0.522	0.318	0.331	0.717	0.669
90-94	0.313	0.519	0.275	0.271	0.753	0.564
95-99	0.314	0.521	0.252	0.234	0.789	0.493
100-104	0.328	0.525	0.242	0.214	0.817	0.443
105-109	0.354	0.530	0.245	0.212	0.838	0.421
110+	0.376	0.536	0.253	0.218	0.853	0.416

Table 14. Alpha values to index family-specific 'levels' by life expectancy at birth, males

	Family				
Target e_0	1	2	3	4	5
30	0.982	1.477	1.505	1.911	0.487
32.5	0.934	1.425	1.461	1.868	0.423
35	0.884	1.371	1.416	1.823	0.352
37.5	0.831	1.314	1.369	1.776	0.272
40	0.774	1.253	1.318	1.727	0.177
42.5	0.713	1.187	1.265	1.675	0.060
45	0.645	1.116	1.208	1.619	-0.070
47.5	0.571	1.039	1.145	1.559	-0.182
50	0.487	0.957	1.076	1.494	-0.290
52.5	0.397	0.867	0.999	1.429	-0.393
55	0.287	0.764	0.920	1.355	-0.495
57.5	0.146	0.645	0.828	1.272	-0.603
60	-0.051	0.506	0.717	1.174	-0.714
62.5	-0.250	0.347	0.580	1.060	-0.828
65	-0.431	0.150	0.408	0.929	-0.947
67.5	-0.611	-0.100	0.162	0.772	-1.075
70	-0.798	-0.381	-0.218	0.565	-1.216
72.5	-0.988	-0.635	-0.525	0.281	-1.364
75	-1.201	-0.885	-0.816	-0.152	-1.526
77.5	-1.425	-1.146	-1.099	-0.506	-1.709
80	-1.667	-1.395	-1.390	-0.842	-1.904
82.5	-1.935	-1.665	-1.697	-1.176	-2.122
85	-2.221	-1.942	-2.005	-1.517	-2.361
87.5	-2.524	-2.236	-2.338	-1.877	-2.622
90	-2.851	-2.540	-2.684	-2.253	-2.906

Table 15. Alpha values to index family-specific 'levels' by life expectancy at birth, females

	Family				
Target e_0	1	2	3	4	5
30	1.022	1.578	1.562	1.952	0.533
32.5	0.979	1.534	1.521	1.916	0.474
35	0.937	1.490	1.481	1.878	0.415
37.5	0.892	1.447	1.441	1.840	0.349
40	0.845	1.401	1.399	1.800	0.273
42.5	0.796	1.354	1.355	1.758	0.183
45	0.742	1.305	1.309	1.715	0.067
47.5	0.684	1.249	1.259	1.668	-0.062
50	0.620	1.190	1.205	1.619	-0.167
52.5	0.549	1.126	1.147	1.565	-0.267
55	0.470	1.056	1.082	1.507	-0.364
57.5	0.383	0.979	1.010	1.448	-0.460
60	0.276	0.897	0.935	1.383	-0.558
62.5	0.130	0.802	0.849	1.308	-0.661
65	-0.078	0.690	0.746	1.223	-0.767
67.5	-0.275	0.557	0.619	1.121	-0.876
70	-0.455	0.399	0.460	1.003	-0.991
72.5	-0.636	0.196	0.249	0.867	-1.119
75	-0.824	-0.087	-0.120	0.691	-1.255
77.5	-1.016	-0.371	-0.452	0.451	-1.402
80	-1.232	-0.638	-0.752	0.074	-1.565
82.5	-1.458	-0.904	-1.044	-0.389	-1.747
85	-1.715	-1.181	-1.355	-0.766	-1.949
87.5	-1.991	-1.472	-1.679	-1.146	-2.177
90	-2.303	-1.788	-2.028	-1.520	-2.434