Extended Abstract – TA MOULTRIE and R van GIJSEN

Background

According to Booth (1979; 1984) and Brass and Airey (1988), the standard designed for use with the Relational Gompertz model was chosen to correct data problems commonly found in high fertility populations. As such, the standard should be particularly useful in Africa and especially sub-Saharan Africa, since this is the region with the highest total fertility rates and arguably the poorest quality data in the world. According to Guengant and May (2001), about a third of African countries were yet to experience large fertility declines by the 1990's.

Evidence from the sub-Saharan Demographic and Health Surveys (DHS) show that, although the TFR is still high relative to developed countries, fertility in the region has indeed declined and is still declining, supporting the findings by Cohen (1993; 1998), Garenne and Joseph (2002) and Caldwell and Caldwell (2002) amongst others.

Fertility is no longer high relative to the levels existing when the standard was derived, and therefore it is pertinent to ask whether the Booth standard, derived to represent mid- to high fertility populations in the 1980s, is still appropriate for use today.

In particular, as already discussed, Booth and others warn that estimates derived using the relational Gompertz model with the Booth standard when the underlying pattern of fertility differs will result in poor and biased estimates. If the parameters fall outside the ranges specified by Zaba (1981) then the assumption that limits the Taylor expansion to two difference terms is violated and the third and subsequent terms of the expansion becomes significant. Certainly, it is already well understood that the parameters α and β must fall within the ranges, $-0.3 < \alpha < 0.3$ and $0.8 < \beta < 1.25$, to avoid omitted terms in the Taylor expansion underlying the Booth standard from becoming significant.

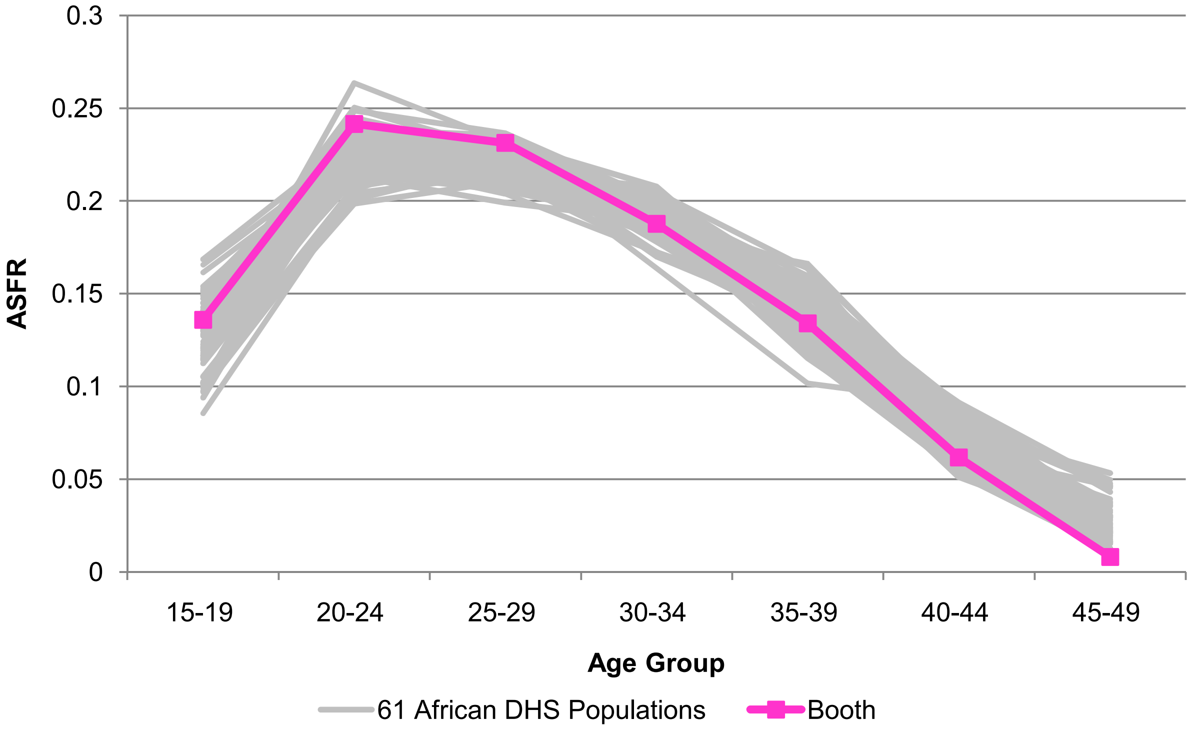
The Booth standard and high fertility

Recent applications of the Relational Gompertz model to African data – see, for example, Moultrie and Timæus (2003); Moultrie and Dorrington (2004); Dorrington, Moultrie and Daniel (2006) for South Africa and Botswana, and Mturi and Hlabana (2000) for applications to Lesotho – have shown a consistent pattern whereby the modelled results in the oldest age group are significantly lower than the rates observed in the data.

For some time, the supposition was that this finding was the consequence of an 'adoption effect' – older women being linked inadvertently at a fieldwork or data processing stage to young children. However, investigation of data from a large number of African DHSs would seem to indicate that the matter is too common, and too generalised, to be the result of systematic data error.

Investigations were undertaken on 61 sub-Saharan Demographic and Health Surveys in order to establish if the standard can be employed in these high fertility populations. Figure 1 compares the standardised age-specific fertility data from the 61 DHS schedules to the Booth standard.

Figure 1: ASFR of the Booth Standard compared to 61 African DHS



Note: ASFRs have been standardised to a TFR of 5.

From Figure 1 one could deduce that the standard is indeed applicable to all high fertility settings. The large number of schedules plotted, however, obscures actual differences. Also, looking at the 45-49 age group it appears that the understatement in older age fertility persists for the sub-Saharan DHSs. The consistent underestimate of 45-49 fertility and the broadly similar patterns of fertility presented in Figure 1 suggest that a distinctive African fertility pattern exists, which can be summarised by using the mean fertility rates for each group to obtain an average age-specific fertility schedule (Table 1and Figure 2).

Table 1: Booth Standard compared to African Pattern

Age Groups	Booth Standard	African Pattern
15-19	0.13584	0.12707

20-24	0.24147	0.22663
25-29	0.23130	0.22020
30-34	0.18757	0.18969
35-39	0.13401	0.13968
40-44	0.06169	0.07053
45-49	0.00812	0.02621

The schedule given in Table 1 also minimises the sum of squared deviations and gives the maximum likelihood estimates for each age group. As a result, it is believed to best represent the pattern of African fertility.

Misfit of the Booth standard to the African pattern

As expected, given that the Booth standard understated 45-49 fertility for the individual DHS schedules, the Booth standard also underestimates the African pattern (Figure 2).

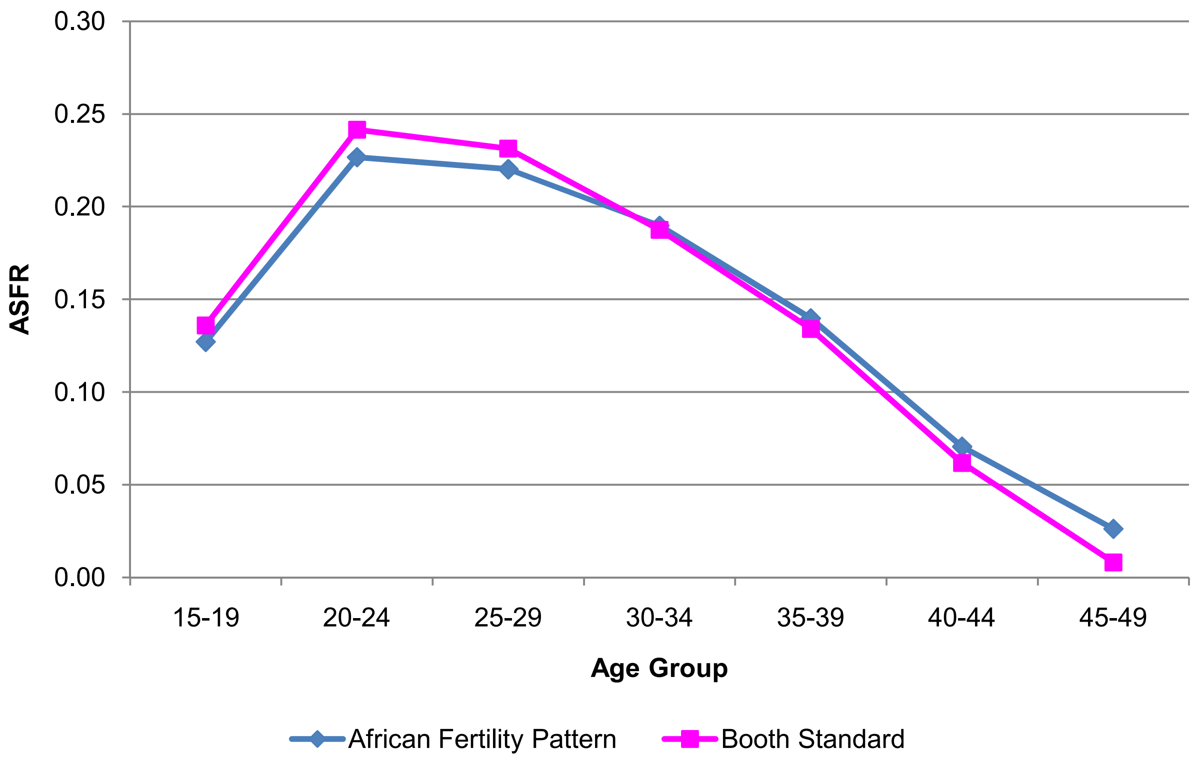


Figure 2: ASFR of the Booth standard compared to the African fertility pattern

In fact, f(45-49) in the Booth standard is only 31 per cent of the average DHS fertility level. This casts serious doubts over the appropriateness of the Booth standard for sub-Saharan African settings.

Source of problems with the schedules

As a result of the misfits observed, the standard must be further investigated to ascertain the reason for the consistent understatement. It is reasonable to begin by interrogating the data and criteria upon which the standard is based. Booth had selected 33 Coale-Trussell schedules (Coale-Trussell, 1974) as the basis for the development of the standard, but recognised the need to increase fertility in the tails of the distribution. Two criteria were set to achieve this - f(10-19) > 0.15 and f(35-49) > 0.21. The former ensures that schedules with high early fertility are used to determine fertility at the youngest ages. Similarly, the latter criterion guarantees that schedules with high old age fertility are used to obtain the standard levels for the older age groups.

These two conditions are compared to the age-group specific fertility rates observed for the 61 DHS populations (Table 2).

Table 2: Evaluation of the Booth Criteria for inclusion in the dataset

	33 Coale-Trussell Schedules		61 DHS Populations	
	f(10-19)	f(35-49)	f(10-19)[1]	f(35-49)
Average	0.14818	0.19701	0.12706	0.23692
Minimum	0.08499	0.15864	0.08549	0.18505
Maximum	0.20747	0.23030	0.16845	0.29814

The table results suggest that the criterion f(10-19) > 0.15 is too high. This is evidenced by Booth schedules with a 17 per cent higher average and approximately 25 per cent higher maximum than the DHS schedules. This higher average level of early fertility will suppress 45-49 fertility since fertility is cumulative and must always reach its maximum by age 50 (the accepted end of the fecund period).

By contrast, the second criterion - f(35-49) > 0.21 - would appear to be too low. As can be seen in the table the maximum of the DHS populations is 44 per cent above the Booth equivalent. The disparity is further emphasised by the fact that even the average of the DHS schedules is higher than the maximum of the 33 Coale-Trussell schedules. Although the criterion states "greater than" - leaving the upper end open - the starting value of 0.21 clearly allows the inclusion of too many low values and, consequently, 45-49 fertility is further restricted.

These results suggest that the problem lies not with the methodology Booth employed, but with the dataset upon which the standard is based. As such, it becomes important to assess the dataset used by Booth. She sets the criteria $10 \leq a_0 \leq 15$, $0.1 \leq k \leq 1$, $0 \leq m \leq 0.6$ and SMAM[2] ≤ 21 to select a sub-set of 33 Coale-Trussell schedules believed to capture high fertility patterns (Booth 1979, p. 49).

[1] The DHS data does not include f(10-14) and it is assumed to be negligible with the result that f(10-14) ≈ 0.0. Consequently, f(10-19) = f(15-19) for these populations.

[2] SMAM is a frequently used abbreviation for the Singulate Mean Age at Marriage.

Table 3 compares the average, minimum and maximum values of fertility for the 45-49 age group.

Table 3: Comparison of the f(45-49) values

	33 Coale-Trussell Schedules	61 DHS Populations
Average	0.00810	0.02621
Minimum	0.00580	0.00949
Maximum	0.01021	0.05324

The table shows that the Coale-Trussell values are significantly lower than the equivalent measures for the 61 surveyed populations. The average and maximum of the sub-Saharan populations are respectively 220 per cent and 420 per cent higher than the equivalents for the data used by Booth. Critically, the maximum of the 33 Coale-Trussell schedules - f(45-49) = 0.01021 - is barely higher than the minimum for the DHS populations - f(45-49) = 0.00949.

Figure 3, similarly, illustrates the disparity between the fertility rates of the 61 DHS surveys and the Booth standard for the 45-49 age group.

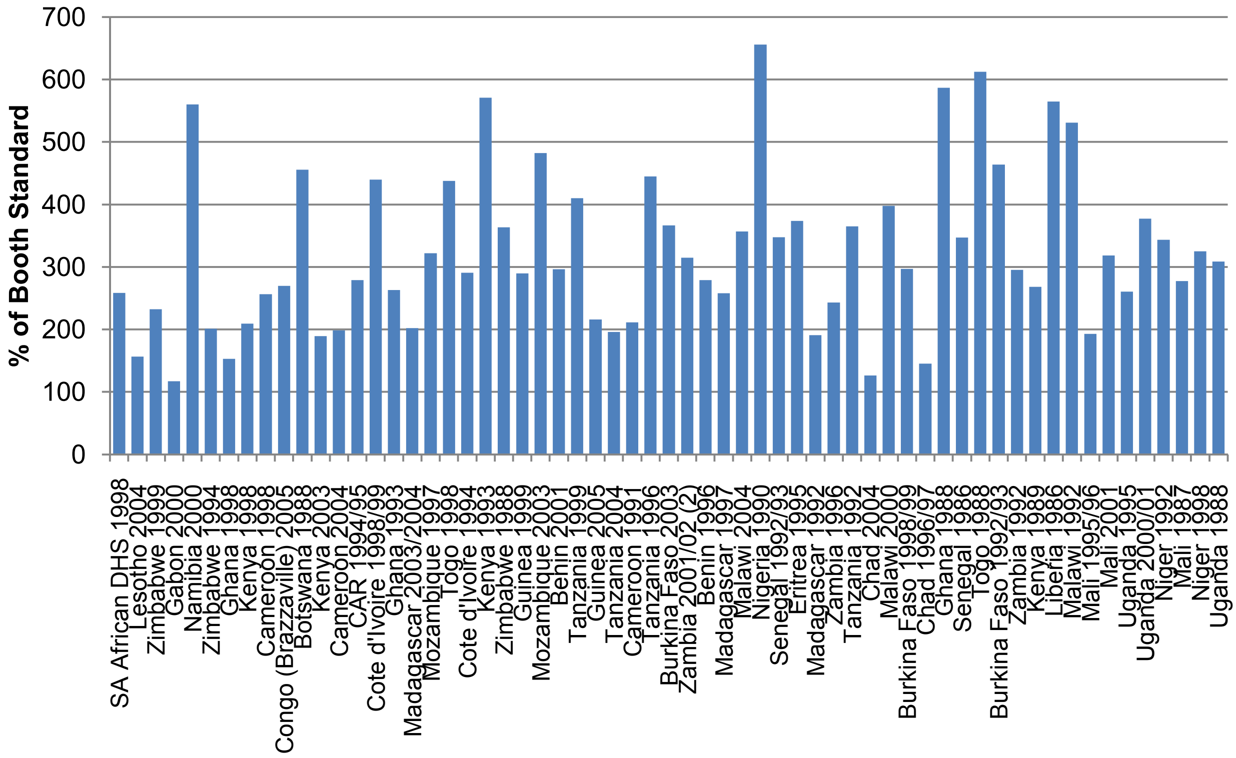


Figure 3: Ratio of the 45-49 fertility of the sub-Saharan DHS to the Booth Standard

Having removed the level effect - by standardising the data to a TFR of five - inspection of Figure 3 shows that the difference between the Booth standard and sub-Saharan fertility rates for the 45-49 age group ranges from 17 per cent (Gabon 2000) to 555 per cent (Nigeria 1990) with an average difference of 224 per cent.

This supports the finding that it is the *pattern* of fertility - and not the fertility level - that determines if a standard is appropriate. In additionone must conclude that the Booth standard does not and cannot capture the effect of old age fertility for these African populations, and that an alternative to the Booth standard is required for African populations.

Alternatives to the Booth standard

Although the preceding material suggests that the Booth standard is not appropriate for use in the analysis of sub-Saharan Africa populations and that the data used by Booth cannot be employed to develop an alternative standard for African settings, the Coale-Trussell model cannot be summarily rejected as a data source based on evidence from only 33 schedules. As a result, the Coale-Trussell model must be reinvestigated to establish if an alternative data set can be found that will yield a standard fertility pattern more appropriate to the African DHS populations. We automated the Coale-Trussell model to produce 64 000 fertility schedules, allowing the value of a_0 to vary within the range [9.25, 19] in increments of 0.25 years. Similarly, the parameters k and m are both allowed to take values between 0.05 and 2 (inclusive) in intervals of 0.05.

Investigations into the DHS data suggested that 61 of 78 schedules examined had the highest fertility between 20 and 29. Just under 24000 (23864) of the Coale-Trussell schedules had the same peak – but every one of these had fertility in the 45-49 age group lower than the average from the DHS data; and the average of the 23864 schedules for this age group was only slightly more than half that observed. Furthermore, of the 64 000 sechdules, that with the highest 45-49 fertility does not reflect the pattern of the African fertility pattern (Figure 3). By contrast, the Coale-Trussell schedule defined by the parameters $a_0 = 12.5$, $k = 0.45$ and $m = 0.15$ provides the closest fit to the African pattern based on minimising the sum of squared error.

However, as seen in Figure 3, despite the remarkably close fit over the ages 15 to 44 a dramatic underestimate is observed for the 45-49 age group. In fact, the best-fitting schedule understates 45-49 fertility by more than 150 per cent.

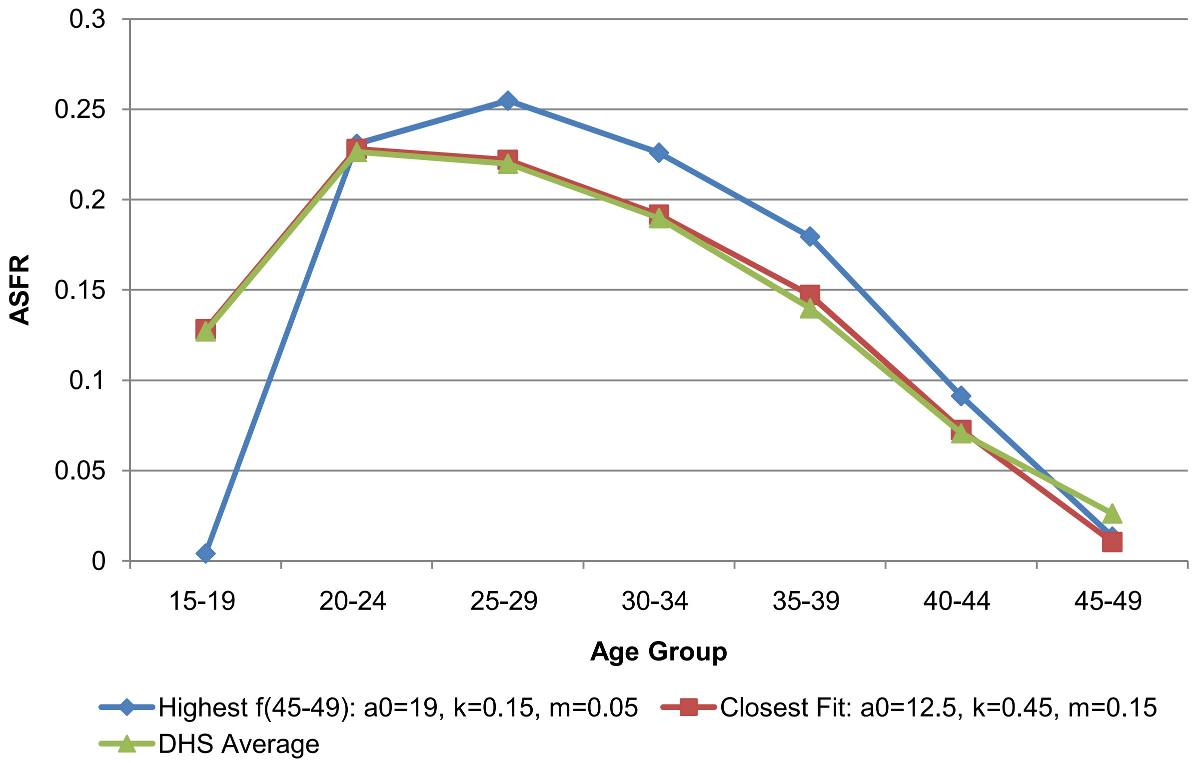


Figure 3: Comparison of average DHS fertility with two Coale-Trussell schedules

However, the Coale-Trussell model was originally conceived as a model of marital fertility. It is therefore necessary to consider the modifications to the model that have been proposed. Xie (1990) and Xie and Pimentel (1992) have already reformulated the Coale-Trussell model in an attempt to maintain its relevance in the face of mounting criticism of its inapplicability to a variety of modern situations. Xie (1990) focuses on adjusting the vector *n(a)* whilst Xie and Pimentel (1992) modify *v(a)*.

Applying the first adjustment (to n(a)) proposed by Xie to the 64 000 schedules resulted in 27144 having the same mode of fertility. Also, as expected given the higher n(45-49), f(45-49) is higher than in the original formulation. the maximum ASFR in the oldest age category has increased by 32 per cent from f(45-49) = 0.0135 to f(45-49) ≈ 0.0178, but still only about 68 per cent of the average African level - f(45-49) = 0.0262. Evidently, the Xie adjustment moves the pattern in the requisite direction, but not sufficiently far.

An alternative modification to the model is that by Xie and Pimentel (1992). Their reformulation restates the model as a statistical method and derives a new series of *v(a)*-values, the parameters governing fertility control in the original model.

Examination shows that these schedules have a maximum f(45-49) of approximately 0.0195 obtained for the schedule given by a_0 = 11, k = 2 and m = 1.3. Despite the improvement of almost 44 per cent over the original 45-49 fertility, this maximum is about 35 per cent below the equivalent African level and the problem clearly persists. In addition, this schedule - like the schedules in sections 4.2 and 4.3.1 - must be rejected for the African populations since it does not sufficiently resemble the African fertility pattern (Figure 4).

As with the two models above, a better overall fit to the African pattern can be found with an alternative parameterisation. The schedule with the best overall fit (i.e. the lowest sum of squared error) is defined by the parameters a_0 = 9.5, k = 0.65 and m = 0.15. However, this schedule reports age-group specific fertility for the 45-49 age group of 0.0151 which is a 72 per cent understatement relative to the average sub-Saharan DHS level.

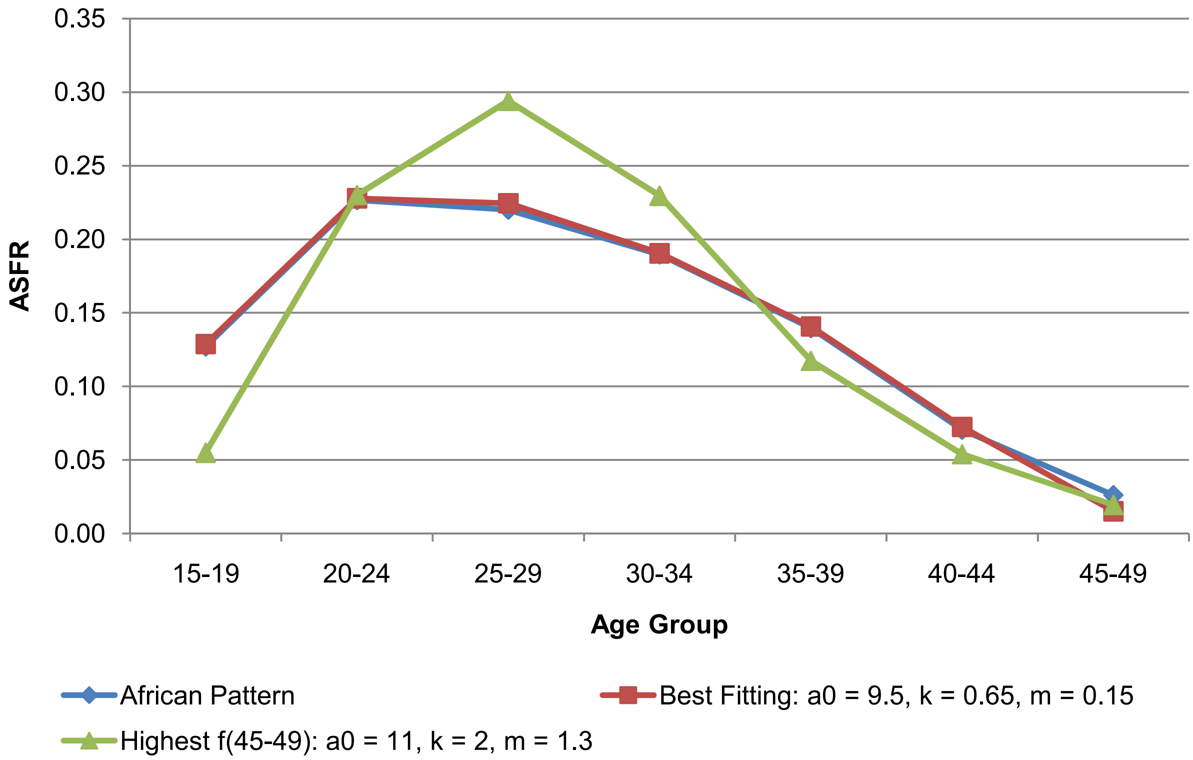


Figure 4: Comparison of the African pattern with two schedules using changed v(a)

Despite the improvements by Xie (1990) and Xie and Pimentel (1992) the model continues to understate fertility in the oldest age group. Although successfully adjusting *r(a)* and increasing fertility in the oldest age group, even the best-fitting schedule understates f(45-49) by more than 70 per cent.

Alternatives to the Coale-Trussell Model

The misfit of 45-49 fertility observed using the Coale-Trussell model necessitate an analysis of alternative methods of representing sub-Saharan Africa data, particularly if a different standard is to be developed.

Hoem et al. (1981) analyse Danish fertility data using a number of different fertility distributions. They conclude that the cubic spline provides the best fit with the Hadwiger function, Gamma density and Coale-Trussell model as joint second best. Furthermore, they show that the Brass polynomial is less accurate (Hoem, et al. 1981).

However, we have already shown that the Coale-Trussell model is not appropriate for the sub-Saharan populations being considered. Gage also dismisses cubic splines for mammalian populations since it "requires good underlying empirical data" (Gage 2001, p.490). Hence, by the same argument cubic splines are dismissed for the sub-Saharan populations due to the enduring problems around data quality.

Gage further shows that the Brass polynomial cannot be rejected in favour of other, more complex, methods (Gage 2001). Consequently, both the Brass polynomial and Hadwiger functions are investigated for the usefulness in measuring and capturing African fertility patterns. These functions will then be evaluated against each other using graduation methods.

Brass polynomial

Gage (2001) has generalised the polynomial given in Brass (1975) as follows:

$$\mathrm{F(z)} = \mathrm{c}\int_{\mathrm{s}}^{\mathrm{z}} (x - s)(s + w - x)^2 \, \mathrm{dx}$$

In order to determine the appropriateness of the Brass polynomial to an African pattern this generalised equation was applied to the DHS data. As before, a macro in MS Excel 2007 was used to automate the production of these schedules. The parameters were allowed to vary such that $9 \leq s < 21$ and $22 \leq w < 52$. This produced 36000 schedules for analysis and the resultant age-specific fertility rates are ranked using the same procedure as before. Investigation shows that 11055 meet the criterion that the sum of the ranks for the 20-24 and 25-29 age groups equals three. The average DHS fertility rates are then compared to the approximately 11000 remaining schedules by minimising the sum of squared differences (SSE).

The best fit is achieved by the schedule with parameters s = 13.2 and w = 38.9 (SSE = 0.00034). However, like the Booth standard and the Coale-Trussell models discussed above, this schedule still understates 45-49 fertility by almost 27 per cent. Although this is an improvement, a closer match to f(45-49) is achieved with the parameters s = 13 and w = 39.8 while maintaining good overall fit (SSE = 0.00040). Table 4 compares the African pattern to the latter model schedule.

Table 4: Model schedule using the Brass polynomial

x	African f(x,x+4)	Model Schedule f(x,x+4)	Percentage Error
15-19	0.12707	0.13285	4.5%
20-24	0.22663	0.20925	-7.7%
25-29	0.22020	0.22360	1.5%
30-34	0.18969	0.19410	2.3%
35-39	0.13968	0.13899	-0.5%
40-44	0.07053	0.07646	8.4%
45-49	0.02621	0.02475	-5.6%

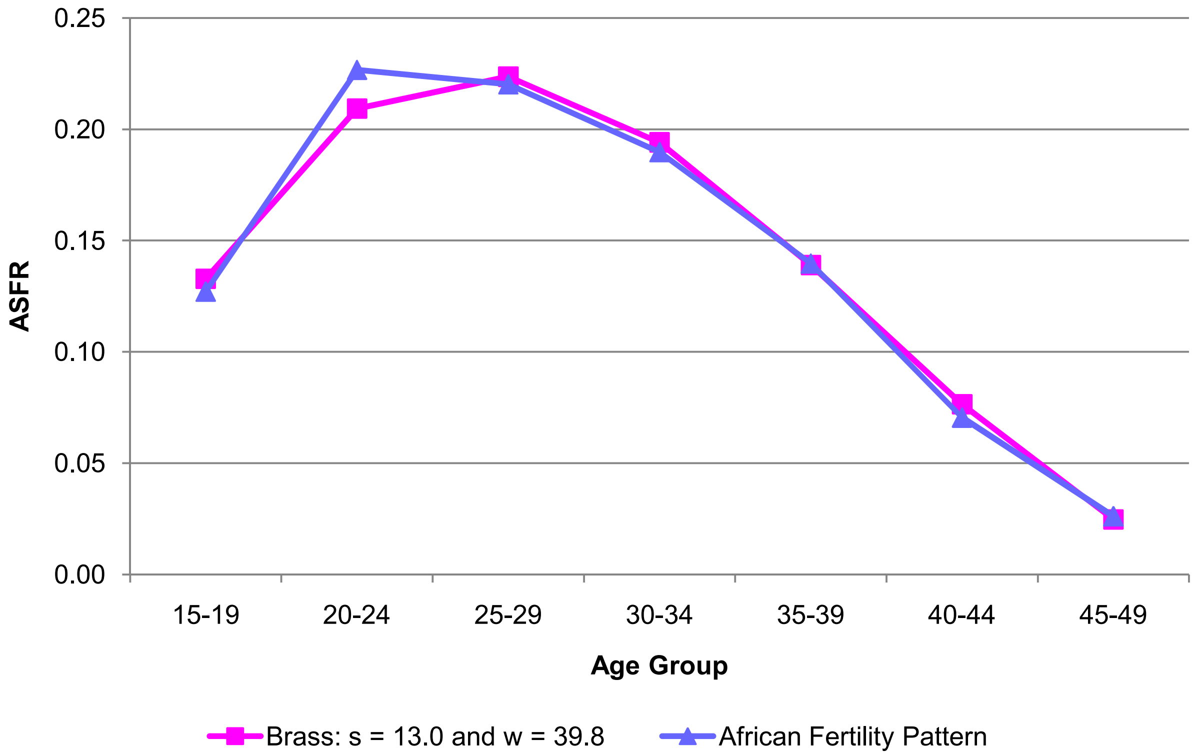


Figure 5: Comparison of ASFR for the African pattern and Brass model schedule

Using the Brass polynomial has reduced the fertility underestimate for the oldest age group to 5.6 per cent. This is a significant improvement over the Coale-Trussell schedules which showed under-estimates of 151, 90 and 72 per cent, respectively, for the original, Xie and Xie and Pimentel formulations discussed earlier.

Hadwiger function

Hoem et al. (1981) and Gage (2001) indicate that a reasonable alternative to the Coale-Trussell model may be the Hadwiger function.

$$m_x = \left(\frac{ab}{c}\right) * \left(\frac{c}{x}\right)^{\frac{3}{2}} * e^{\left[-b^2 * \left(\frac{c}{x} + \frac{x}{c} - 2\right)\right]}$$

Despite having two parameters, b and c, with no clear demographic interpretation and being more complex than the Brass polynomial, the Hadwiger function must be considered. The reason for this is that both Hoem et al. and Gage show that the Hadwiger function consistently provides higher estimates of 45-49 fertility than the Coale-Trussell model.

As with the Brass polynomial and Coale-Trussell models a procedure was set up in MS Excel to automate the production of fertility schedules using the Hadwiger function. Since the investigation is concerned with pattern rather than level the total fertility parameter, a, is given a value of one. The remaining parameters – b, c, s and w – were allowed to take values within predefined ranges: $1.7 \leq b \leq 3.4$ with step-size 0.1, $23 \leq c \leq 34$ in steps of 0.25, $47 \leq w \leq 53$ where w takes only integer values and $11 \leq s \leq 17$ at half-year ages. However, analysis showed that no additional benefit is gained by including half-year ages for s or incrementing c by 0.25.

As a result, the parameter c was set to increase by steps of 0.5 and starting age, s, takes on integer ages. This process results in 20286 schedules to be compared with the sub-Saharan Africa experience and the average fertility derived from the 61 DHS populations. Of these schedules, 10731 satisfied the ranking criterion that fertility must be highest between ages 20 and 30.

A number of Hadwiger schedules may be used to describe the average DHS fertility pattern. Some of these schedules slightly overstate and some understate 45-49 fertility. However, the best fitting schedule is defined by the parameters $b = 1.9$, $c = 31$ and $17 \leq x \leq 47$. That is, x starts at age $s = 17$ and ends at the maximum $w = 47$. Table 5 and Figure 6 compare this model schedule to the DHS average age-group specific fertility rates.

Table 5: Model schedule using the Hadwiger function

x	DHS Average f(x,x+4)	Model Schedule f(x,x+4)	Percentage Error
15-19	0.12707	0.13020	2.5%
20-24	0.22663	0.22570	-0.4%
25-29	0.22020	0.22538	2.4%
30-34	0.18969	0.18143	-4.4%
35-39	0.13968	0.12872	-7.8%
40-44	0.07053	0.08440	19.7%
45-49	0.02621	0.02417	-7.8%

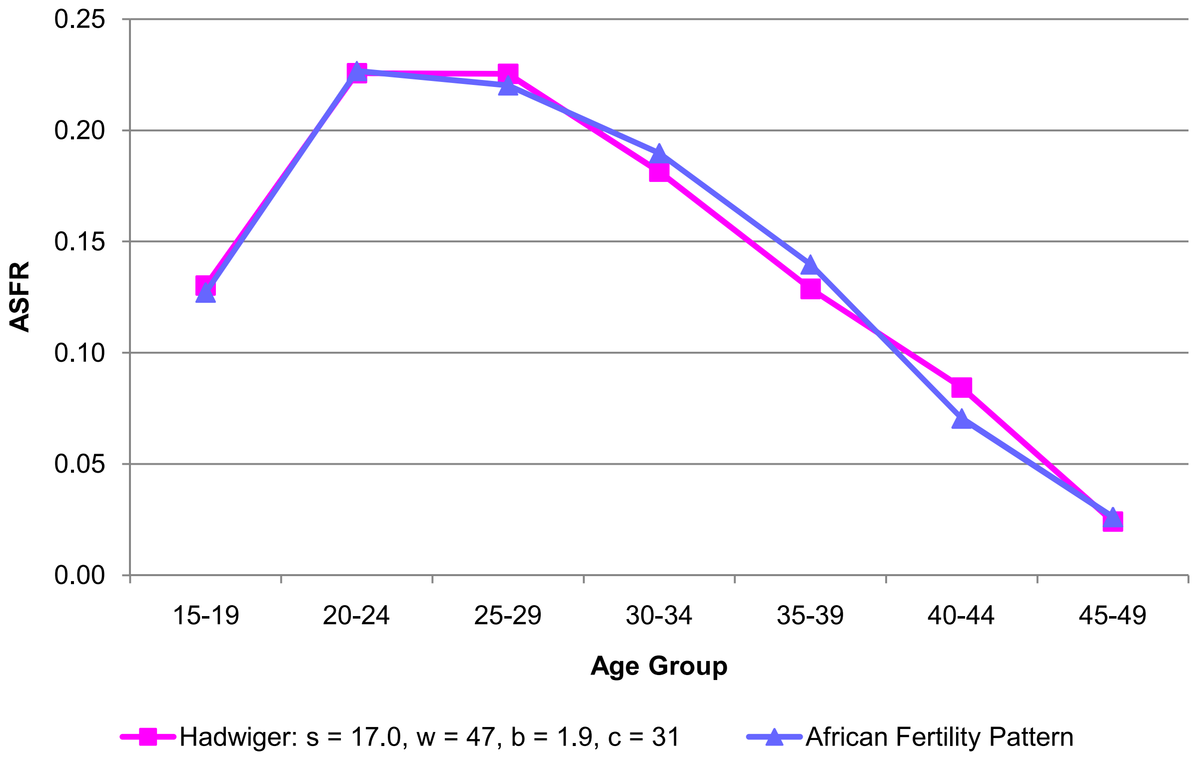


Figure 6: ASFR of African pattern compared to Hadwiger model schedule

As with the Brass polynomial, there is an improvement in the estimates of f(45-49) on those obtained from the Coale-Trussell schedules discussed in sections 4.1 and 4.2. The Hadwiger-based standard understates 45-49 fertility by 7.8 per cent and has SSE = 0.00042.

Comparing the Brass Polynomial and Hadwiger Function

Both the Brass polynomial and the Hadwiger function show improvements over the Coale-Trussell based models. Seeing as both methods yield reasonable results a decision must be made on which method to use.

The Brass polynomial has two mathematical advantages over the Hadwiger function. First, the polynomial requires one fewer parameter and the parameters have clear demographic interpretations - c is a measure of TFR whilst s is the starting age and w the length of the fertility period.

Second, the Brass cumulative fertility function can be simplified using the properties of integrals. This allows the direct calculation of cumulative and age-specific fertility rates once s and w are set. By contrast, the Hadwiger function cannot be explicitly evaluated and numerical methods or statistical packages are required to obtain the cumulative and age-specific fertility rates.

Although both these arguments favour the Brass polynomial neither is sufficient to dismiss the Hadwiger function. As such, graduation tests must be conducted on both standards to consider both smoothness and goodness-of-fit to the African fertility pattern. The results of the goodness-of-fit tests are summarised in Table 6.

Table 6: Comparison of Brass and Hadwiger standards with the African fertility pattern

Age Group	African ASFR	Hadwiger standard	Signs of Differences	Brass standard	Signs of Differences
15-19	0.12707	0.13020	+	0.13285	+
20-24	0.22663	0.22570	-	0.20925	-
25-29	0.22020	0.22538	+	0.22360	+
30-34	0.18969	0.18143	-	0.19410	+
35-39	0.13968	0.12872	-	0.13899	-
40-44	0.07053	0.08440	+	0.07646	+
45-49	0.02621	0.02417	-	0.02475	-
SSE (15-49)		0.00042		0.00040	
SSE (35-49)		0.00032		0.00004	

Both tests show good overall fit to the data as a result of minimising the squared difference terms. The Brass polynomial has lower SSE than the Hadwiger function over the entire age range. The second test is for consistent over- or underestimation identified by excessive numbers of deviations with the same sign. For both standards there is no reason evidence of a consistent under- or overestimation since both have three deviations of one sign and four with the other sign. The third check tests for correlation between deviations and looks for runs of the same sign. Again, both fertility schedules give no evidence to indicate correlation between differences.

Despite meeting the goodness-of-fit criterion both schedules must still be tested for smoothness. Smoothness is defined as smooth third differences. Based on this measure of smoothness the standard derived using the Brass polynomial is smooth whereas the Hadwiger-based standard is not. As such, the Hadwiger schedule is dismissed based on lack of smoothness.

In addition, although both series have good overall fit, it is the fit in the older age groups that has consistently been the problem. Critically, the Brass polynomial fits the older age groups better than the Hadwiger function as illustrated by the lower SSE for the 35-49 age group.

So, in terms of simplicity, goodness-of-fit and smoothness the Brass polynomial (with parameters s = 13 and w =39.8) yields a better standard for the African pattern than the Hadwiger function.

The African Standard and the Relational Gompertz Model Coefficients

The chosen standard was derived to assist in the analysis of African fertility data and, in particular, when using the relational Gompertz model. The relational Gompertz model requires cumulative fertility rates without a half-year shift, cumulative fertility with a half-year shift and average parities (Table 7). In addition, the single-year standardised age-specific fertility rates of this standard are presented in Table 8.

Table 7: P(i), f(x, x+4) and F(x) for standard excluding the 10-14 age group

Age	F(x) without shift	F(x) with shift	i	Age Group	f(x,x+4)	P(i)
20	0.13285	0.11535	1	15-19	0.13285	0.07548
25	0.34210	0.31974	2	20-24	0.20925	0.24084
30	0.56570	0.54395	3	25-29	0.22360	0.45302
35	0.75980	0.74232	4	30-34	0.19410	0.66640
40	0.89879	0.88740	5	35-39	0.13899	0.83427
45	0.97525	0.96998	6	40-44	0.07646	0.94186
50	1.00000	0.99903	7	45-49	0.02475	0.99087

Table 8: Single-year ASFR for standard excluding the 10-14 age group

Age	f(x)	Age	f(x)	Age	f(x)
15	0.01687	27	0.04507	39	0.02277
16	0.02237	28	0.04445	40	0.02021
17	0.02721	29	0.04350	41	0.01768
18	0.03141	30	0.04226	42	0.01521
19	0.03499	31	0.04076	43	0.01282
20	0.03799	32	0.03903	44	0.01055
21	0.04043	33	0.03709	45	0.00842
22	0.04235	34	0.03497	46	0.00647
23	0.04377	35	0.03271	47	0.00472
24	0.04472	36	0.03033	48	0.00320
25	0.04524	37	0.02786	49	0.00194
26	0.04534	38	0.02533		

The standard rates presented above are applicable to populations, like the 61 sub-Saharan DHS, where no data are available for the 10-14 age group. By contrast to the DHS data, some fertility data includes the 10-14 age group. The inclusion of this age group means that the standard rates above cannot be used without the loss of potentially valuable information about early fertility.

The tables below give the cumulative fertility rates, age-specific fertility rates and parity for the standard when data are available for the 10-14 age group. As before, the Brass polynomial with parameters s = 13 and w = 39.8 is used, which ensures consistency between the standards.

Table 9: P(i), f(x, x+4) and F(x) for standard including the 10-14 age group

Age x	F(x) without shift	F(x) with shift	i	Age Group	f(x,x+4)	P(i)
15	0.01417	0.00892	0	10-14	0.01417	0.01202

20	0.14514	0.12789	1	15-19	0.13097	0.08858
25	0.35142	0.32938	2	20-24	0.20628	0.25160
30	0.57185	0.55041	3	25-29	0.22043	0.46250
35	0.76321	0.74597	4	30-34	0.19135	0.67113
40	0.90022	0.88900	5	35-39	0.13702	0.83662
45	0.97560	0.97040	6	40-44	0.07538	0.94268
50	1.00000	0.99904	7	45-49	0.02440	0.99100

Table 10: Single-year ASFR for standard including the 10-14 age group

Age	f(x)	Age	f(x)	Age	f(x)
10	0	24	0.04409	37	0.02746
11	0	25	0.0446	38	0.02497
12	0	26	0.0447	39	0.02245
13	0.00367	27	0.04443	40	0.01992
14	0.01051	28	0.04382	41	0.01743
15	0.01663	29	0.04288	42	0.01499
16	0.02206	30	0.04166	43	0.01264
17	0.02683	31	0.04018	44	0.01040
18	0.03096	32	0.03847	45	0.00830
19	0.03449	33	0.03656	46	0.00638
20	0.03745	34	0.03447	47	0.00465
21	0.03986	35	0.03224	48	0.00315
22	0.04175	36	0.0299	49	0.00192
23	0.04315				

The relational Gompertz model requires the calculation of the coefficients e(x) and g(x) from the cumulative fertility rates (with and without ½-year shift). Similarly, the average parities require that e(i) and g(i) be calculated. Table 11 lists the coefficients of the relational Gompertz model associated for the African pattern where the 10-14 age group is excluded.

Table 11: Standard Relational Gompertz model parameters (10-14 age group excluded)

	No Shift			½-year Shift			Parity	
Age	e(x)	g(x)	Age	e(x)	g(x)	Age Group	e(i)	g(i)
20	1.4750	-1.4193	19 ½	1.4651	-1.4844	15-19	1.5321	-1.6807
25	1.4741	-0.7869	24 ½	1.4825	-0.8502	20-24	1.5475	-1.0884
30	1.3374	-0.1166	29 ½	1.3570	-0.1888	25-29	1.4214	-0.4694
35	1.0924	0.6915	34 ½	1.1239	0.5991	30-34	1.2418	0.2513
40	0.7123	1.7930	39 ½	0.7625	1.6569	35-39	0.9480	1.1615
45	0.0000	3.6865	44 ½	0.1459	3.3772	40-44	0.5054	2.4758
						45-49	0	4.6917

For the standard including the 10-14 age group, the model coefficients are recalculated and given in Table 12.

Table 12: Standard Relational Gompertz model parameters (10-14 age group included)

No Shift			½-year Shift			Parity		
Age	e(x)	g(x)	Age	e(x)	g(x)	Age Group	e(i)	g(i)
15	1.2603	-2.1046	14 ½	1.2138	-2.1932	10-14	1.0628	-2.645
20	1.5052	-1.3822	19 ½	1.4999	-1.4444	15-19	1.2897	-1.7438
25	1.4837	-0.764	24 ½	1.4931	-0.8265	20-24	1.4252	-1.0157
30	1.341	-0.0985	29 ½	1.361	-0.1703	25-29	1.3726	-0.3355
35	1.0937	0.7074	34 ½	1.1254	0.6152	30-34	1.1421	0.4391
40	0.7127	1.8079	39 ½	0.7629	1.6719	35-39	0.7061	1.5117
45	0.0001	3.701	44 ½	0.1459	3.3917	40-44	0.2765	3.2104
						45-49	0	6.0547

Comparison of Relational Gompertz Results from the Booth and African Standards

The final test of the revised African standard is to apply the relational Gompertz model - with the African standard - to actual data from sub-Saharan populations. The results can then be compared to those obtained using the Booth standard. Two populations - Kenya 1979 and Botswana 2001 census - are assessed.

Table 13 shows the original DHS data for Kenya 1979 and the standardised results obtained from the Booth and African standards. The last column of the table shows the percentage difference between the Booth estimates and the African estimates.

Table 13: Standardised relational Gompertz model results for Kenya DHS 1979

Age Group	Original	Booth	African	% Difference
15-19	0.08416	0.10957	0.11865	8%
20-24	0.22580	0.22991	0.21251	-8%
25-29	0.23589	0.23629	0.23417	-1%
30-34	0.19329	0.19960	0.20139	1%
35-39	0.14771	0.14656	0.13942	-5%
40-44	0.07757	0.06885	0.07253	5%
45-49	0.03558	0.00923	0.02133	131%

As can be seen from Table 13 the Kenyan data exhibits the classic, African uptick in 45-49 fertility. A look at the estimates derived using the Booth and African standard show that the Booth standard dramatically understates f(45-49) by almost 75 per cent. By contrast, the estimate based on the African standard is much higher at 60 per cent of the observed level. This constitutes a 35 per cent reduction in the understatement and an increase of 131 per cent over the Booth estimate for the oldest age group.

Table 14, in an analogous fashion to Table 13, presents the data for Botswana 2001 as well as the relational Gompertz estimates derived using the Booth and African standards.

Table 14: Standardised relational Gompertz model results for Botswana census 2001

Age Group	Original	Booth	African	% Difference
15-19	0.07812	0.09800	0.11080	13%
20-24	0.22759	0.21085	0.19101	-9%
25-29	0.20620	0.22887	0.21944	-4%
30-34	0.19616	0.20532	0.20289	-1%
35-39	0.16087	0.16147	0.15410	-5%
40-44	0.09459	0.08284	0.09010	9%
45-49	0.03647	0.01266	0.03164	150%

As with the Kenyan data, the Botswana data exhibits high fertility in the 40-44 and, particularly, 45-49 age groups. The African standard again yields estimates of 45-49 fertility that are markedly higher than those obtained when using the Booth standard while still maintaining comparable levels over the remainder of the age range. In fact, the estimates of f(45-49) based on the Booth standard are only about 35 per cent of the observed levels in the Botswana population. By contrast, the estimates derived from the African standard are 150 per cent higher than those derived using the Booth standard and at about 87 per cent of the observed rate.

For both surveys the relational Gompertz model using the Booth standard understates fertility in the oldest age group. As explained in previous chapters this results not from Booth's methodology, but from the data upon which the standard is based. By contrast, the model using the African standard provides estimates more representative of the African fertility pattern and, consequently, displays higher estimates of 45-49 fertility than are observed when using the Booth standard.

We therefore recommend the adoption of the parameters above for use with the relational gompertz model to estimate fertility in African populations.

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