Cause-Decomposition of the Change over Time in the Modal Age at Death

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Abstract

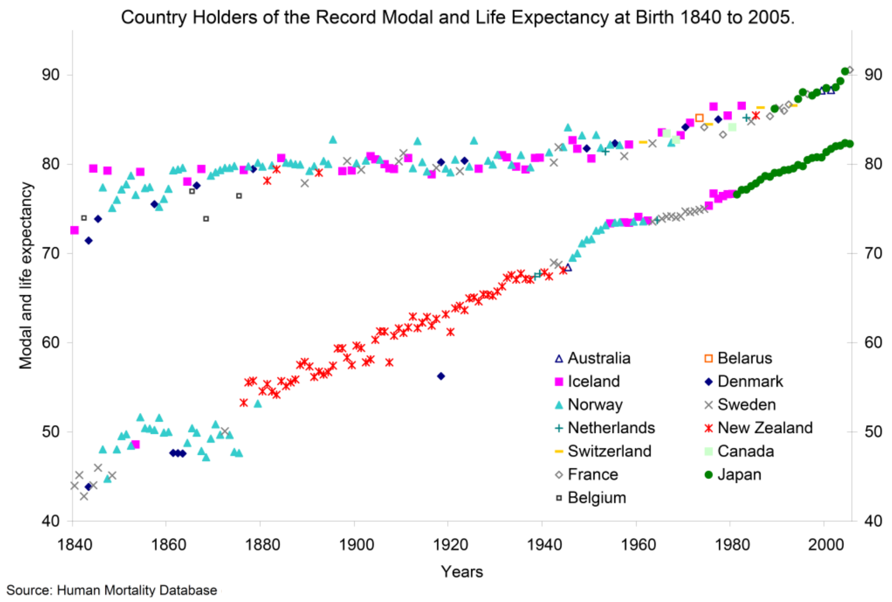
The great shift in causes of mortality and morbidity that occurred in the developed world predominantly during the twentieth century is recognized in the framework of the demographic and epidemiologic transitions. Today's industrialized countries have largely progressed through both transitions, as observed in the uninterrupted long historical rise in life expectancy at birth. This paper discusses the methodology to study the cause-contribution of specific causes of death to the change in measures of longevity: The mean and modal ages at death. Methods to assess the cause-decomposition of the change in life expectancy, or mean age at death, are widely used by demographers. Less attention has been put to study methods of cause-contribution to the change in modal age at death, or age with the highest probability of dying. The main aim of this paper is to present a new proposal of cause-decomposition to the change in modal age at death. This decomposition is based on the recent finding that for the modal age at death to change toward older ages it is required that the decline in mortality occurs at ages older than the modal age at death. The method follows two simple steps: (i) to identify modal ages at two given years *M(t1)* and *M(t2), for* years *t1* and *t2* respectively*;* (ii) For each year *t1* and *t2* the cause of death distribution after the two modal ages is used to apply the statistical calculations of composition analysis of causes of death. This method allows studying the cause of death change that went along the decline in death rates and which triggered the increase in the modal age at death.

Introduction

The aging of populations is a fact that is observed in many countries: in an accelerated rise of the numbers of older aged (85 and over) persons [20], an increase in the numbers and percentages of centenarians and supercentenarians [17], a steady rise in the maximum recorded life span [22], an uninterrupted long historical rise in life expectancy [13,21]. These facts have contributed to an increased interest in the ancient question on the possible causes of long life.

Currently, mortality is concentrated at older ages in many countries. The age where most of the deaths occur, modal age at death also referred to as the mode, is seldom proposed as a measure to study longevity. The life table mean age at death, or life expectancy, is generally preferred. The different trends over time of these two measures correspond to profound changes in mortality reductions over age and causes of death in the last centuries. Figure 1 shows the record life expectancy and modal age at death from 1840 to 2005.

Figure 1



As observed in Figure 1, the record mode stagnated at levels around age 80 with minor increases until the 1960s. During this time record life expectancy increased rapidly from a minimum of 42 in 1840s to age 74 in 1960s. Since then the two measures have increased at a similar pace keeping an average gap of almost ten years from mode to life expectancy. The trends over time of these two longevity measures correspond to population level changes from a dominance of child mortality reductions to a dominance of adult mortality reductions [5]. Studying the modal age at death complements the changes in life expectancy and facilitates the examination of changes in mortality at older ages [4,5,6,7,8,9,18]. In this paper we formulate a first attempt to disentangle the change over time in the modal age at death and compare it to results for the changes observed in life expectancy.

A very useful tool to study change in mortality is decomposition analysis. In general, to decompose means to separate something into its constituent parts or elements or into simpler compounds. Decomposition analysis is a powerful tool to study the effects that each component has on the whole dynamic. A special decomposition analysis that has captured the attention of many demographers is the study of the trends of mortality by examining the change in life expectancy over time, between populations or sexes. The first decompositions of life expectancies were based on the difference in life expectancies [2,15,16]. The work by the mathematical demographer Nathan Keyfitz (1968) on the change over time in life expectancy looks instead at the continuous change of life expectancy. Similar procedures have followed more recently to analyze the impact by causes of death to the change in life expectancy at birth [3,11,19].

However, none of the above methods can be applied to the change over time in the modal age at death. Horiuchi and colleagues mention : “[our] proposed method can be applied not only to the mean but also to the median, the standard deviation, and other summary measures, such as the interquartile range. (However, it would not be appropriate to apply this method in a similar way to the modal age at death, which is not a differentiable function of age-specific death rates.)…” Horiuchi et al. (2009:790).

The aim of the present study is to show the first steps to address this missing methodology in the field and be able to answer: what is the cause of death contribution of the change in the modal age at death? How are these contributions different to what we observed in cause-decomposition of changes in life expectancy?

Methods and Data

<u>Brief Description of the Data Used in this Analysis</u>

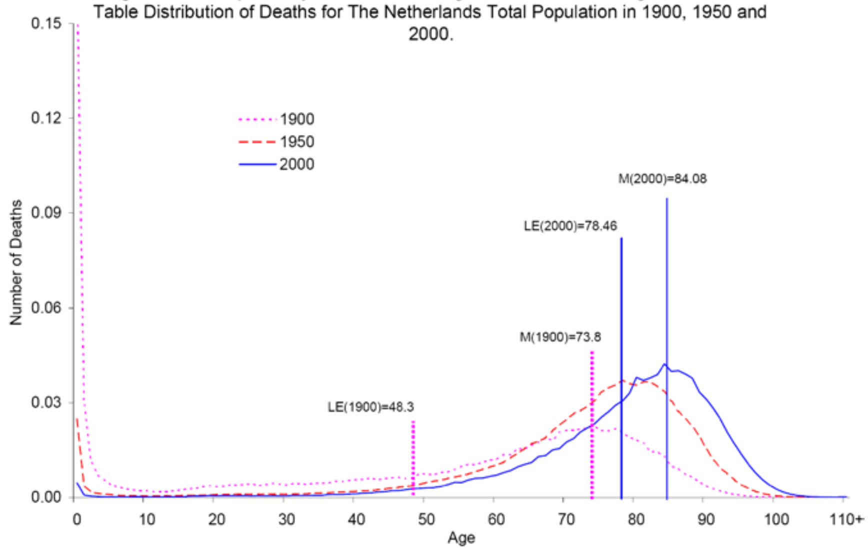
The World Health Organization database contains available information on cause of death for the second half of the twentieth century for country members. From this database we extracted the number of registered deaths by cause (death statistics are coded according to the 7th to 10th revision of the International Classification of Diseases (ICD)), sex and age-group (0, 1, 2, 3, 4, 5-9, ..., 80-84 and age 85 and over) for the 1950s to the first years of the new century. For the purpose of this analysis, we have aggregated causes of death into major categories where the principal criterion for these grouping is etiological similarities. Initially we work with twelve causes of death, but we will expand to a greater number: (1) respiratory tuberculosis, (2) other infectious and parasitic disease, (3) neoplasms, (4) cardiovascular disease, (5) influenza pneumonia and bronchitis, (6) diarrhea, gastritis, enteritis, (7) Certain degenerative disease, (8) complications of pregnancy, (9) certain disease of infancy, (10) motor vehicle accidents, (11) other accidents and violence and (12) with a group of "Remaining causes" for the remaining causes not accounted previously.

The second group of data sources are the Human Mortality Database (HMD) and the Berkeley Mortality Database (BMD). The HMD project contains detailed time series of mortality data and life tables for populations with virtually complete registration and census data. Annual period life tables were extracted from this database. BMD was replaced by the HMD, except for the detailed time series of mortality data by cause of death for Japan which is only available in the BMD.

Brief Description of the Modal Age at Death.

Let the force of mortality at age *a* and time *t* be denoted as $\mu(a,t)$ and let $\ell(a,t)$ be the survival function in the life table population at age *a* and time *t*. The function describing the distribution of deaths (i.e., life spans) for a given age is found as the product of the survival function up to that age multiplied by its force of mortality, $d(a,t)=\mu(a,t)\ell(a,t)$. Figure 2 shows the distribution of life table deaths for the Netherlands at the beginning, middle, and end of the twentieth century.

Figure 2. Life Expectancy, LE, and Modal Age at Death, M, after Age 5 in the Life Table Distribution of Deaths for The Netherlands Total Population in 1900, 1950 and 2000.



Source: Author's calcualtions of M as suggested by Kannisto (2001), based on the Human Mortality Database

Also included in Figure 2 are the life expectancy at birth and modal age after age 5 in 1900 and 2000. Here it is possible to appreciate how life expectancy is found at age 48.3 in 1900, while most of the deaths in this year are concentrated at ages below 5 and around the late modal age of 73.8. In 2000, life expectancy reached a value of 78.5 years, and reduced markedly its distance to the modal age, which moved to age 84. The change from a dominance of child mortality reductions to a dominance of adult mortality reductions causes the change in the life expectancy increase over time [4,5]. In low mortality countries the modal age at death can be an important reference point to study deviations in mortality not perceived in the life expectancy change.

Brief Description of Composition Analysis

In the 2008 PAA meeting Jim Oeppen presented a new idea entitled: "Coherent Forecasting of Multiple-Decrement Life Tables: A Test Using Japanese Cause of Death Data". His suggestion of forecasting cause of death data is strongly based on the analysis used in Geology known as "Compositional Data Analysis or CDA" developed by *Aitchison (1986).* In general, a total (unit in a simplex space in CDA) might be broken down into mutually exclusive and exhaustive parts

$$C = (c^1, c^2, ..., c^n), \qquad (1)$$

which can be standardized to add to one, $\sum_{i=1}^{n} c^i = 1$. Here and in the rest of the text a upper right subscripts 1, 2,…,n denote a partition of the total. Compositional analysis allows comparing between different compositions or comparing changes in one composition over time. For example, the relation between any two compositions C_1 and C_2 can always be expressed as a perturbation operation $(C_1 \Theta C_2)$, which is calculated as

$$(C_1 \Theta C_2) = \left[\frac{c_1^1}{c_2^1}, \frac{c_1^2}{c_2^2}, ..., \frac{c_1^n}{c_2^n}\right] \Big/ \left[\frac{c_1^1}{c_2^1} + \frac{c_1^2}{c_2^2} + ... + \frac{c_1^n}{c_2^n}\right], \qquad (2)$$

where a lower right subscript 1 or 2 denotes the two compositions. This operator returns a partition of the relation of the elements of the two compositions and adds to one, two properties that are crucial in comparing any type of "distance" measure between two compositions. The perturbation

$(C_1 \Theta C_2)$ can be thought as the changes that have to be imposed on $C_1 = (c_1^1, c_1^2, ..., c_1^n)$ to obtain $C_2 = (c_2^1, c_2^2, ..., c_2^n)$ at a later time.

Modal Age at Death Decomposition

The overall trend in mortality in the last 150 years of demographic transition has been a decline (with flu epidemics, war and other sporadic events that have caused some fluctuations in mortality but that have not stopped the overall declining trend). In a situation when only mortality declines are present and changes in mortality occur only at ages younger than the modal age then there is no change in the mode, which remains at its value. However, when changes in mortality take place at ages older than the mode, this could make the modal age change to older ages. We have formally/mathematically shown this in an article on a similar topic [5]. More specific, for a mode at time 1, denoted M_1, to change to an older age at time 2, $M_2 > M_1$, enough reductions in mortality have to occur after the age M_1 to obtain more deaths at M_2 at time 2, $d_2(M_2) > d_2(M_1)$. This age range is the key interval of changes in mortality for studying the change in the modal age at death and can be denoted mathematically as

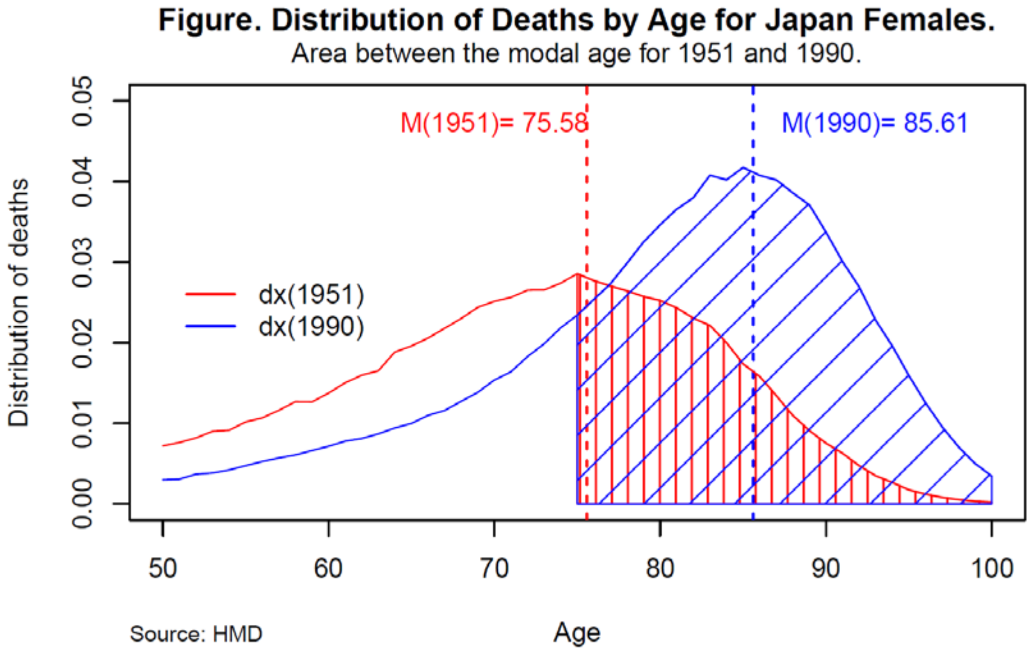
$$d(M_1+) = \int_{M_1}^{\omega} d(x)dx, \qquad (3)$$

where ω is the last age attained by a person in the population. As an illustration of this, Figure 3 shows graphically the age range of interest in the study of the change in the modal age at death for Japan where it change from 1951 with a modal age of M_1=75.58 to 1990 when the modal age at death reached the level of M_2=85.61 (shaded area in Figure 3).

To account for the cause of death contribution to the changes in the modal age we study a function of the difference between the ages $M_2 - M_1$. Assuming that there are $i = 1,2,...,n$ independent causes of death, $d^i(x)$, then the function describing the distribution of deaths at each age is equal to the addition of the number of deaths by the different causes,

$$d(x) = \sum_{i=1}^{n} d^i(x). \qquad (4)$$

Figure 3.



Substituting the latter relation in equation (4) in the integral in equation (3) above returns the cause of death distribution in this age range,

$$d(M_1+) = \sum_{i=1}^{n} \int_{M_1}^{\omega} d^i(x)dx . \tag{5}$$

The compositional data analysis (CDA) can now be used to compare the death distribution changes from time 1 to 2 that is from $d_1(M_1+)$ to $d_2(M_1+)$. Analogous to equations (1) and (2) for the CDA here we use the following partitions

$$c^i = \frac{\int_{M_1}^{\omega} d^i(x)dx}{d(M_1+)} , \tag{6}$$

which assures us that we have the closure $\sum_{i=1}^{n} c^i = 1$. Let finally the decomposition of the change in the modal age at death be calculated as

$$M_2 - M_1 = [M_2 - M_1]\ (C_2 \Theta C_1) . \tag{7}$$

Preliminary Results

Figures 4a and 4b show the cause-decomposition for the change over time in life expectancy and modal age at death in the decades of the 1950s and 1980s respectively. The sensitivity of Life expectancy to specific cause of deaths is evident from these two Figures, while in 1950s almost the entire change in life expectancy is due to infectious diseases by 1980s the improvements in this measure are resultant in great part of cardiovascular diseases reductions. For the modal age death on the other hand there is no single prominent cause of death that accounts for most of the change.

Figure 4a.

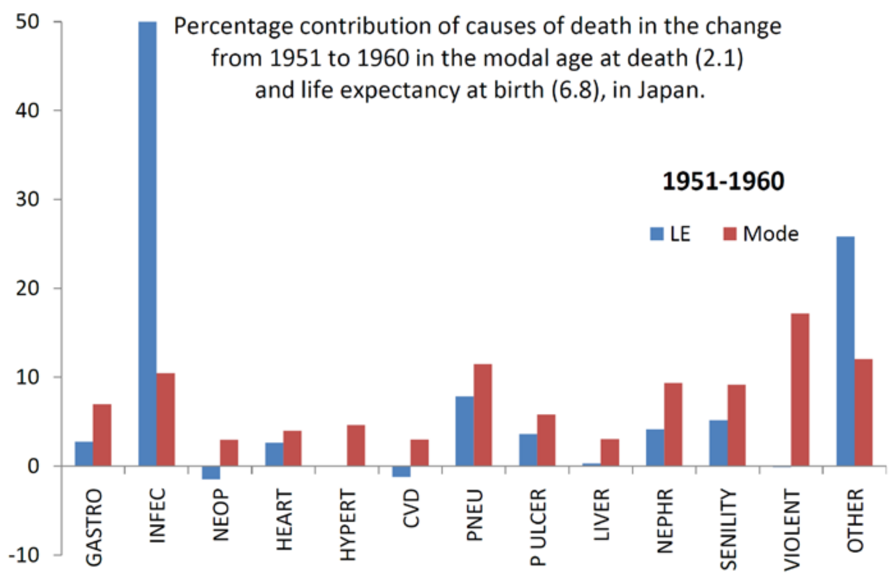
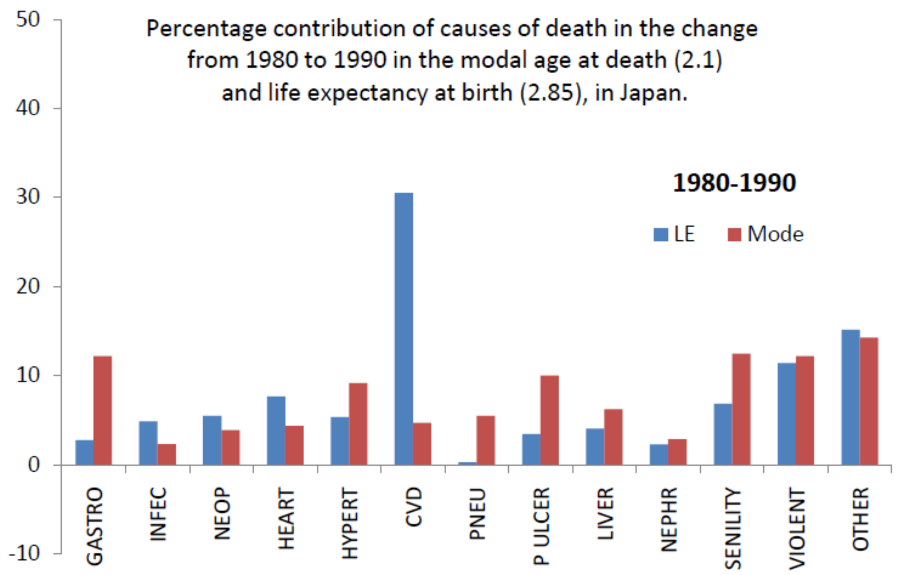


Figure 4b.



Discussion and Future Steps

The appealing sense of the cause-decomposition in equation (7) is that the cause-specific components add to the difference between the two modal ages at time 1 and time 2. Although not shown here, it can be proved that similar types of calculations are carried for the life expectancy cause-decomposition. This latter comparison will be prepared in the finalized version for the PAA meeting in 2011. Furthermore, more current illustrations will be presented using data from WHO member countries, both developed and developing, which will lead to insights on the advantage of having not only the life expectancy cause-decomposition at hand but also the analogous calculations for the change over time in the modal age at death.

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